

UNIT

9

Integers



Pacific



Mountain

Canada has 6 time zones.

- What time is it where you are now?
- You want to call a friend in Newfoundland.

What time is it there?

- In the province or territory farthest from you, what might students be doing now?

What other questions can you ask about this map?



What You'll Learn

- Use integers to describe real-life situations.
- Model integers with coloured tiles.
- Compare and order integers.
- Add integers.
- Subtract integers.

Why It's Important

- We use integers to compare temperatures with the temperature at which water freezes.
- Integers extend the whole number work from earlier grades.





Central



Eastern



Atlantic



Newfoundland



Key Words

- positive number
- negative number
- integer
- positive integer
- negative integer
- opposite integers
- zero pair

Skills You'll Need

Mental Math Strategies for Addition and Subtraction

Example

Add or subtract.

Use mental math.

a) $38 + 17$

b) $111 - 64$

Solution

a) $38 + 17$

Make a friendly number. Subtract from one number and add to the other.

$$\begin{aligned} \text{So, } 38 + 17 &= \underbrace{38 + 2} + \underbrace{17 - 2} \\ &= 40 + 15 \\ &= 55 \end{aligned}$$

Subtract 2 from 17.

Add 2 to 38.

b) $111 - 64$

Subtract the ones, then subtract the tens.

Subtract 4: $111 - 4 = 107$

Then subtract 60: $107 - 60 = 47$

So, $111 - 64 = 47$

Count back by 1s

to subtract 4.

✓ Check

Use mental math.

1. Add or subtract. What strategy did you use?

a) $22 + 88$

b) $69 - 29$

c) $93 + 38$

d) $132 - 85$

2. In a magic square every row, column, and diagonal has the same sum.

Copy and complete each magic square. Explain how you did it.

a)

	1	6
	5	7
4		2

b)

17	10	
	14	
13	18	

We measure temperature in degrees Celsius. The highest temperature in Prince George, BC, on April 27, 2004, was 12°C .

12 is a **positive number**.

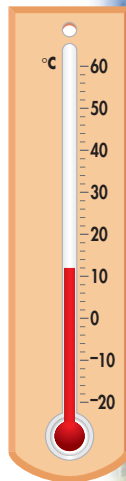
12°C is the difference between the temperature of the air and the temperature at which water freezes, 0°C .

We may not be able to use a positive number to express a difference.

For example, the temperature inside a freezer is 4°C less than the freezing point of water. We represent this difference as -4°C .

-4 is a **negative number**.

We say “negative 4.”



When we talk about temperatures below zero, we say “minus 4,” rather than “negative 4.”

Explore



Work with a partner.

Use a positive or a negative number to represent each situation.

- eight degrees above zero
- ten degrees below zero
- parking three levels below ground level
- two points ahead in a soccer game
- a loss of sixteen dollars
- taking four steps backward

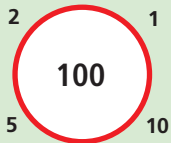
Reflect & Share

Compare your answers with those of another pair of classmates. For each situation, how did you decide whether to use a positive number or a negative number?

Connect

Mental Math

Use any math operation. Use the numbers around the circle. Write as many ways as you can to make the number in the circle.

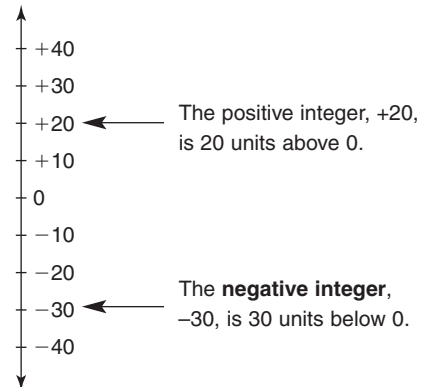


Positive and negative whole numbers are called **integers**.

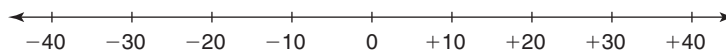
We put $+$ in front of a number to indicate it is a **positive integer**.

We can show integers on a number line.

The number line may be vertical, like a thermometer.



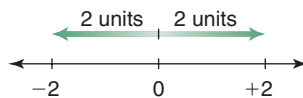
A number line may be horizontal.



Opposite integers are the same distance from 0 but are on opposite sides of 0.

For example, $+2$ and -2 are opposite integers.

They are the same distance from 0 and are on opposite sides of 0.



$+4$ and -4 are also opposite integers, as are $+21$ and -21 .

Practice

- Mark each integer on a number line.
 - $+1$
 - -5
 - -2
 - $+9$
- Write the opposite of each integer.
 - $+3$
 - -1
 - -8000
 - $+10$
- Use a positive or negative integer to represent each situation.
 - thirty-five degrees Celsius below zero in Yellowknife
 - a weather balloon 28 000 m above Earth's surface
 - diving 35 m below the ocean's surface
 - earning \$500
- Use each integer below to describe a situation.
 - $+4$
 - -5
 - $+120$
 - -8500

Use question 3 as a model.

5. Describe two situations in which you might use negative and positive integers. Write integers for your situations.



6. **Assessment Focus** Statistics Canada reported these data about Canada's population.



Years	Births	Deaths	Immigration	Emigration
1961-66	2 249 000	731 000	539 000	280 000
1996-2001	1 704 000	1 095 000	1 051 000	270 000

- Which numbers can be represented by positive integers? Negative integers? Explain.
- For each column of data in the table, find the difference in the numbers. Write the difference as a positive or negative integer. Explain your choice of integer.
- Choose one time period. Use a number line and integers to explain the relationship between births and deaths.
- Choose one time period. Use integers to explain the relationships between immigration and emigration.

Take It Further

7. Changes in stock prices on the Stock Exchange are written as positive or negative integers.
- Express each change as an integer.
 - The value of Apple Computers increased \$2.
 - Palm Tech dropped from \$25 to \$22.
 - MDS started the day at \$13 and ended the day at \$12.
 - Steve bought Global stock at \$10 and sold it for \$15.
 - Look at the two stock prices in part a, ii. How are the prices related?
 - Look at the two stock prices in part a, iv. How are the prices related?
 - How can an integer be used to show the relationship between two prices of a stock? Explain.

Find the price of a stock in the financial section of a newspaper. Follow the price every day for one month. How are integers used to show how the stock price changes?

Reflect

Suppose you read a situation that can be described with integers. What clues do you look for to help you decide whether to use a positive or negative integer? Use examples in your explanation.

Focus Use a number line to order and compare integers.

Elevation is the height above or below sea level. Elevation influences climate and how people live. For example, crops will not grow at elevations above 5300 m.



Explore

Work with a partner.

You will need an atlas or Internet access.

Here are some examples of extreme elevations around the world:

Vinson Massif, Antarctica	4897 m above sea level
Dead Sea, Israel/Jordan	411 m below sea level
Bottom of Great Slave Lake	458 m below sea level
Mt. Nowshak, Afghanistan	7485 m above sea level
Challenger Deep, Pacific Ocean	10 924 m below sea level

Research to find at least 4 more extreme elevations in Canada.

Two should be above sea level, and two should be below sea level.

Order *all* the elevations from least to greatest.

Reflect & Share

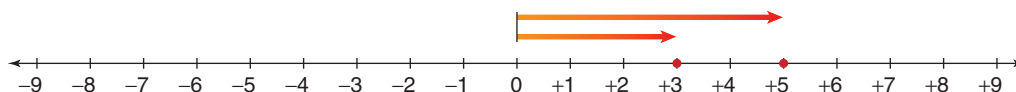
What strategies did you use to order the elevations?

What other ways could you display these data to show the different elevations?

Connect

We use the symbols $>$ and $<$ to show order.

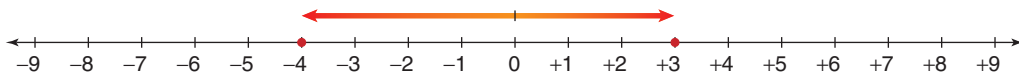
The symbol points to the lesser number.



$+5$ is to the right of $+3$ on a number line.

$+5$ is greater than $+3$, so we write: $+5 > +3$

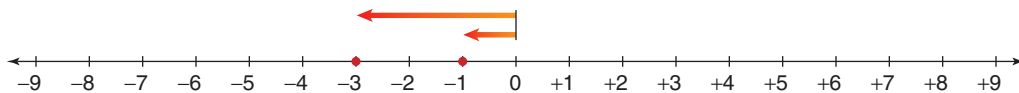
$+3$ is less than $+5$, so we write: $+3 < +5$



+3 is to the right of -4 on a number line.

+3 is greater than -4 , so we write: $+3 > -4$

-4 is less than $+3$, so we write: $-4 < +3$



-3 is to the left of -1 on a number line.

-3 is less than -1 , so we write: $-3 < -1$

-1 is greater than -3 , so we write: $-1 > -3$

Example

For the integers $0, +1, -2, +3, -5$:

- Use a number line to order the integers from least to greatest.
- Write the opposite of each integer.
Show the opposites on a number line.
- Order the opposites from least to greatest.

Solution

- Mark each integer on a number line.



The integers increase from left to right.

So, the integers from least to greatest: $-5, -2, 0, +1, +3$

- The opposite of an integer is its mirror image reflected in a vertical line drawn through 0 on a number line.

The opposite of -5 is $+5$. The opposite of -2 is $+2$.

Since 0 is the reference point, 0 is its own opposite.

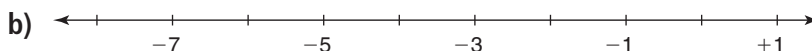
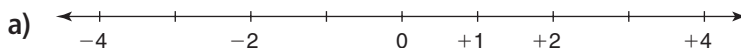
The opposite of $+1$ is -1 . The opposite of $+3$ is -3 .



- So, the opposite integers from least to greatest: $-3, -1, 0, +2, +5$

Practice

1. Copy each number line. Fill in the missing integers.



2. Order the integers in each set from least to greatest.

- a) +5, +13, +1 b) -3, -5, -4 c) +4, -2, +3

3. Order the integers in each set from greatest to least.

- a) +4, +1, +8 b) -7, -5, -3 c) 0, +4, -4

4. Order the integers in each set from least to greatest.

- a) +5, -5, +4, +2, -2 b) -8, -12, +10, 0, -10
c) +41, -39, -41, -15, -25 d) +1, -1, +2, -2, +3

5. Order the integers in each set from greatest to least.

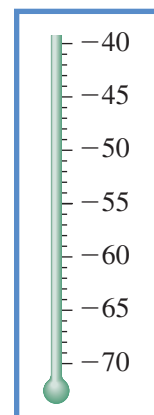
- a) +14, -25, -30, +3, -10 b) 0, +1, +2, -1, -2
c) -29, +27, -11, -4, +6 d) -7, +8, -9, +10, -11

6. This table shows the coldest temperatures ever recorded in 6 provinces and territories.

a) Draw a thermometer like the one shown.

Mark each temperature on it.

Province or Territory	Coldest Temperature °C
Alberta	- 61
Manitoba	- 53
Nova Scotia	- 47
Nunavut	- 64
Ontario	- 58
Quebec	- 54



A real thermometer does not show negative signs. We include them for better understanding.

b) Order the temperatures in part a from least to greatest. How can you use your thermometer to do this?

7. Copy and complete by placing $<$ or $>$ between the integers. Use a number line if it helps.

- a) $+5 \square +10$ b) $-5 \square -10$ c) $-6 \square 0$
d) $-5 \square -4$ e) $+100 \square -101$ f) $-80 \square -40$

Calculator Skills

Tina bought 6 packages of markers for her friends. Each package cost \$4.95. The cashier charged 15% tax. Tina left the store with \$13.42. How much money did Tina start with?

8. Assessment Focus

Look at the integers in the box.

	+4	
-8		-5
	+9	
0		+8

- a) Which integers are:
- greater than 0?
 - between -3 and $+3$?
 - greater than -10 and less than -5 ?
 - less than $+1$?
- b) What other questions can you ask about these integers? Write down your questions and answer them.

9. On January 18, 2002, the temperature in Charlottetown, Prince Edward Island, was -21°C ; in Sydney, Nova Scotia, it was -23°C ; in Point Lepreau, New Brunswick, it was -22°C . Which place was the warmest? Coldest? How do you know?

Take It Further

10. a) Draw a number line from -6 to $+6$. Find the integer that is:
- halfway between -6 and $+6$
 - halfway between -5 and $+1$
 - halfway between -5 and -1
 - 1 less than $+3$
 - 3 more than -4
 - 4 less than -1
- b) Explain why the answer for part a, ii is different from the answer for part a, iii.
- c) -3 is halfway between two integers on a number line. Draw a number line and mark the two integers on it.

11. One day, the temperature in Wabash Lake, Newfoundland, was -41°C ; in Pelly Bay, Nunavut, it was -51°C . The temperature in Churchill, Manitoba, was halfway between these temperatures. What was the temperature in Churchill?

12. Copy each pattern. Extend the pattern for 3 more terms. Describe each pattern in words.

- a) $-5, -3, -1, +1, \dots$ b) $+7, +4, +1, -2, \dots$
c) $-20, -18, -16, -14, \dots$ d) $-5, -10, -15, -20, \dots$

Reflect

When two integers have different signs, how can you tell which is greater? When two integers have the same sign, how can you tell which is greater?

9.3

Representing Integers

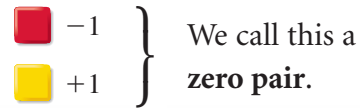
Focus Use coloured tiles to represent integers.

We can use coloured tiles to represent integers.

One yellow tile  can represent +1.

One red tile  can represent -1.

A red tile and a yellow tile combine to model 0:



Explore



Work with a partner.

You will need coloured tiles.

How many different ways can you use coloured tiles to model each number?

➤ 0, +1, +2, +3, +4

➤ -1, -3, -5, -6

Draw a picture to show the tiles you used for each way you found.

Reflect & Share

Compare your models with those of another pair of classmates.

Look at all the models that represent one integer.

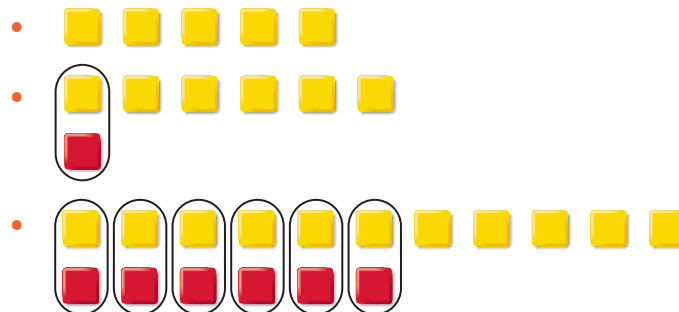
How do you know that all the models represent that integer?

Connect

We can model any integer in many ways.

Each set of tiles below models +5.

Each pair of 1 yellow tile and 1 red tile makes a zero pair. The pair models 0.



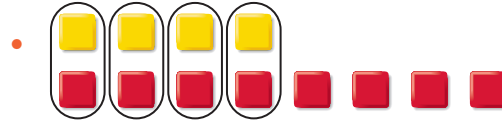
Example

Use coloured tiles to model -4 three different ways.

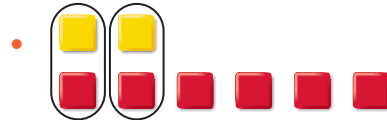
Solution

Start with 4 red tiles to model -4 .
Add different numbers of zero pairs.
Each set of tiles below models -4 .

Adding 4 zero pairs does not change the value.



Adding 2 zero pairs does not change the value.

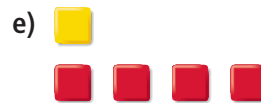
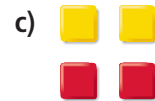
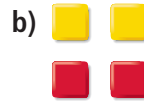


Adding 7 zero pairs does not change the value.



Practice

1. Write the integer modelled by each set of tiles.



2. Use yellow and red tiles to model each integer. Draw the tiles.

a) -6

b) $+7$

c) $+6$

d) -2

e) $+9$

f) -4

g) 0

h) $+10$

3. Work with a partner. Place 10 yellow and 10 red tiles in a bag.

a) Pull out a handful of tiles.

Tell the integer that the tiles model.

b) Have your partner tell what other set of tiles could model this integer.



Mental Math

Sarah ran a marathon in 3 h and 41 min. The race started at 7:45 a.m. At what time did Sarah finish?

4. Assessment Focus

- Choose an integer between -9 and $+6$.
Use coloured tiles to model the integer.
- How many more ways can you find to model the integer?
Create a table to order your work.
- What patterns can you find in your table?
- Explain how the patterns in your table can help you model an integer between -90 and $+60$.

- Suppose you have 10 yellow tiles.
How many red tiles would you need to model $+2$?
How do you know?
 - Suppose you have 100 yellow tiles.
How many red tiles would you need to model $+2$?
How do you know?

Math Link

Sports

In golf, a hole is given a value called **par**.

Par is the number of strokes a good golfer takes to reach the hole.

A score of $+2$ means a golfer took 2 strokes more than par, or 2 strokes over par.

A score of -1 means a golfer took 1 stroke fewer than par, or 1 stroke under par.

Some scores have special names.

A score of $+1$ is a bogey.

A score of -1 is a birdie.

A score of -2 is an eagle.

In a golf tournament, the golfer with the fewest strokes wins the game.

Reflect

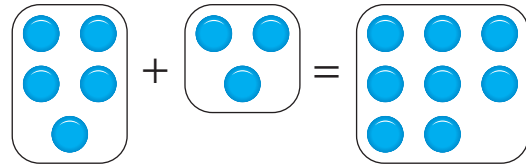
Explain how it is possible to use different sets of red and yellow tiles to model the same integer.

9.4

Adding Integers with Tiles

Focus Use coloured tiles to add integers.

Recall that when you add two numbers, such as $5 + 3$, you combine 5 counters with 3 counters to obtain 8 counters.



You can add two integers in a similar way. You know that $+1$ and -1 combine to make a zero pair. We can combine coloured tiles to add integers.

Explore



Work with a partner.

You will need coloured tiles.

- Choose two different positive integers. Add the integers. Draw a picture of the tiles you used. Write the addition equation.
- Repeat the activity for a positive integer and a negative integer.
- Repeat the activity for two different negative integers.

Reflect & Share

Share your equations with another pair of classmates. How did you use the tiles to find a sum of integers? How can you predict the sign of the sum?

Connect

- To add two positive integers: $(+5) + (+4)$
Model each integer with tiles.

+5:

+4:

This is an addition equation. →

Combine the tiles. There are 9 yellow tiles.

They model +9.

$$\text{So, } (+5) + (+4) = +9$$

- To add a negative integer and a positive integer: $(-6) + (+9)$
Model each integer with tiles. Circle zero pairs.



There are 6 zero pairs.

There are 3 yellow tiles left.

They model +3.

$$\text{So, } (-6) + (+9) = +3$$

- To add two negative integers: $(-3) + (-7)$
Model each integer with tiles.



Combine the tiles. There are 10 red tiles.

They model -10.

$$\text{So, } (-3) + (-7) = -10$$

Example

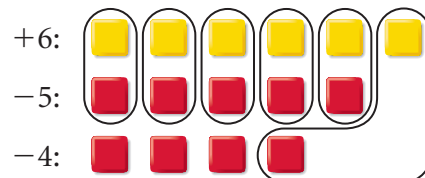
Add. $(+6) + (-5) + (-4)$

Solution

$(+6) + (-5) + (-4)$

Model each integer with tiles.

Circle zero pairs.



There are 3 red tiles left.

They model -3.

$$\text{So, } (+6) + (-5) + (-4) = -3$$

Practice

Use coloured tiles.

1. What sum does each set of tiles model?

Write the addition equation.



2. What sum does each set of tiles model?

How do you know you are correct?

- a) 3 yellow tiles and 2 red tiles
 b) 3 yellow tiles and 4 red tiles
 c) 2 red tiles and 2 yellow tiles

3. Use coloured tiles to represent each sum. Find each sum.

Sketch the tiles you used.

- a) $(+2) + (-2)$ b) $(-4) + (+4)$ c) $(+5) + (-5)$

4. Add. Sketch coloured tiles to show how you did it.

- a) $(+2) + (+3)$ b) $(-3) + (+4)$ c) $(-4) + (-1)$
 d) $(+1) + (-1)$ e) $(-3) + (-4)$ f) $(+5) + (-2)$

5. Add.

- a) $(+4) + (+3)$ b) $(-7) + (+5)$ c) $(-4) + (-5)$
 d) $(+8) + (-1)$ e) $(-10) + (-6)$ f) $(+4) + (-13)$

6. Represent each sentence with integers, then find each sum.

- a) The temperature is -3°C and rises 4 degrees Celsius.
 b) Ravinder earned \$5 and spent \$3.

7. Write 3 integer addition problems.

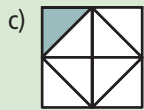
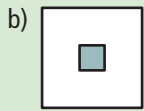
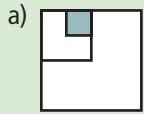
Trade problems with a classmate.

Solve your classmate's problems with coloured tiles.



Number Strategies

About what fraction of the large square is shaded in each diagram?



8. Add. Sketch coloured tiles to show how you did it.

a) $(+1) + (+2) + (+3)$ b) $(+2) + (-1) + (+3)$

c) $(-3) + (-1) + (-1)$ d) $(+4) + (-3) + (+1)$

e) $(-4) + (+1) + (-2)$ f) $(-5) + (-3) + (-2)$

9. Copy and complete.

a) $(+5) + \square = +8$ b) $\square + (-3) = -4$ c) $(+3) + \square = +1$

d) $(-5) + \square = -3$ e) $(+2) + \square = +1$ f) $\square + (-6) = 0$

10. Assessment Focus

a) Add: $(+3) + (-7)$

b) Suppose you add the integers in the opposite order: $(-7) + (+3)$. Does the sum change?

Use coloured tile drawings and words to explain the result.

c) How is $(-3) + (+7)$ different from $(+3) + (-7)$? Explain.

11. In a magic square every row, column, and diagonal has the same sum. Copy and complete each magic square. How did you do it?

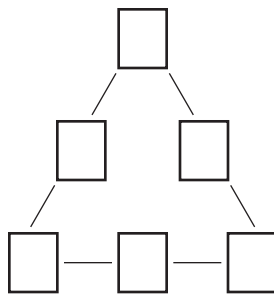
a)

+3		+1
	0	
-1		

b)

-1		+1
	-2	
		-3

Take It Further



12. Add.

a) $(+4) + (+1) + (+1) + (+1)$

b) $(-3) + (+2) + (-1) + (+1)$

c) $(+3) + (-2) + (-1) + (-1)$

13. Copy the triangle at the left.

Place the numbers $-6, -5, -4, -3, -2, -1$ in the triangle so the sums of the sides are the same.

Explain how you solved the problem.

Reflect

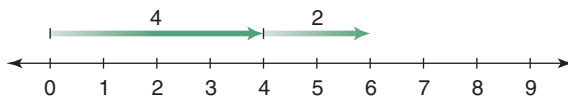
Explain how to add two integers when the integers have:

- the same signs
- opposite signs

Focus Add integers using number lines and patterns.

We can show the addition of whole numbers on a number line.

$$4 + 2 = 6$$

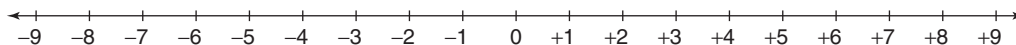


We can also show the addition of integers on a number line.

Explore

Work with a partner.

You will need copies of this number line.



- Choose two different positive integers.
Use a number line to add them.
Write the addition equation.
- Repeat the activity for a positive integer and a negative integer.
- Repeat the activity for two different negative integers.

Reflect & Share

Trade your addition problems with another pair of classmates.

Answer your classmates' problems.

Compare answers.

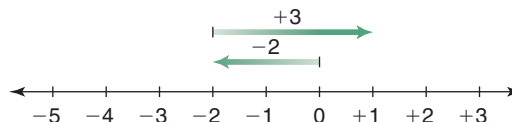
If the answers do not agree, decide who is correct. Explain.

Connect

Start at -2 .

Move 3 units right.

- To add a positive integer, move right (in the positive direction).
 $(-2) + (+3)$
 $(-2) + (+3) = +1$

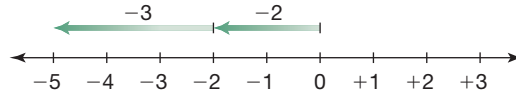


Start at -2 .
Move 3 units left.

➤ To add a negative integer, move left (in the negative direction).

$$(-2) + (-3)$$

$$(-2) + (-3) = -5$$



We can use patterning to add integers.

Example 1

Add. $(+5) + (-2)$

Solution

$$(+5) + (-2)$$

Think of a pattern. Start with facts you know.

Continue the pattern until you reach the fact you want.

$$(+5) + (+2) = +7$$

$$(+5) + (+1) = +6$$

$$(+5) + (0) = +5$$

$$(+5) + (-1) = +4$$

$$(+5) + (-2) = +3$$

Example 2

Sandra and Joe buy and sell CDs at a flea market.

In August, they bought 14 CDs for \$5 each.

They sold 6 CDs for \$9 each.

a) Did Sandra and Joe make money or lose money in August?

b) How much money did they make or lose?

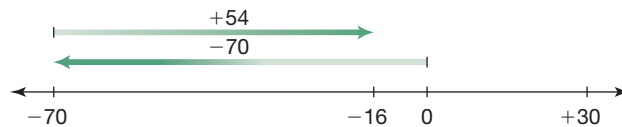
Solution

$$\text{Expenses: } 14 \times \$5 = \$70$$

$$\text{Income: } 6 \times \$9 = \$54$$

a) Since the expenses are greater than the income, Sandra and Joe lost money in August.

b) Draw a number line.



$$(-70) + (+54) = -16$$

In August, Sandra and Joe lost \$16.



Practice

Use a number line when it helps.

1. Add.

a) $(+1) + (+3)$

b) $(-1) + (+3)$

c) $(-3) + (+1)$

d) $(-1) + (-3)$

2. Find each sum. Use a pattern when you can.

a) $(+4) + (+2)$

b) $(+5) + (-3)$

c) $(-4) + (-2)$

d) $(-8) + (+2)$

3. a) Reverse the order of the integers in question 2, then add.

b) Compare your answers to the answers in question 2.

What do you notice?

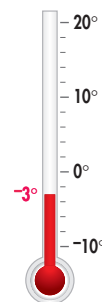
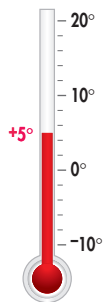
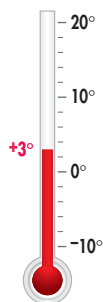
c) Make a general statement about your observations.

4. Look at these thermometers. Find each temperature after:

a) it falls 4°C

b) it falls 7°C

c) it rises 6°C



5. a) The temperature rises 7°C , then drops 2°C .

What is the overall change in temperature?

b) Adrian loses \$4, then earns \$8.

Did Adrian gain or lose overall?

c) The value of a stock went up \$3, then down \$2.

What was the final change in the value of the stock?

6. Add.

a) $(+12) + (+3)$

b) $(+13) + (-7)$

c) $(-5) + (-10)$

d) $(-5) + (+8)$

e) $(-8) + (-7)$

f) $(+4) + (-10)$



Mental Math

Which expression does not belong? Why?

$12 \times 4 \times 50$

$25 \times 10 \times 10$

24×100

$99 \times 20 + 420$

80×30

$90 \times 30 - 300$



Take It Further

7. Add.
- a) $(+30) + (+10)$ b) $(+20) + (-10)$
c) $(-35) + (-5)$ d) $(-15) + (+18)$
e) $(-82) + (+79)$ f) $(-58) + (-22)$
8. **Assessment Focus** Use an example to explain why each statement is true.
Use a number line to support your explanations.
- a) The sum of two opposite integers is 0.
b) The sum of two positive integers is always positive.
c) The sum of two negative integers is always negative.
d) The sum of a negative integer and a positive integer is sometimes negative and sometimes positive.
9. Add.
- a) $(+4) + (+3) + (-6)$ b) $(-2) + (-4) + (+1)$
c) $(-5) + (+3) + (-4)$ d) $(+6) + (-8) + (+2)$
e) $(+12) + (-3) + (-2)$ f) $(-5) + (-8) + (-10)$
10. The temperature in North Bay is 23°C .
The temperature falls 7°C , then rises 12°C .
What is the final temperature?
How did you find out?
Which model did you use?
11. Susanna earned \$24, spent \$7, earned \$12 more, and spent \$10 more. Express her earnings and spendings using integers.
How much money does Susanna have left over from her earnings and spendings?
12. Copy and complete.
- a) $(+10) + \square = +25$ b) $(-10) + \square = -25$
c) $(+20) + \square = +15$ d) $(-20) + \square = -15$
e) $(+35) + \square = +17$ f) $(-35) + \square = -17$

Reflect

You have used three models to add integers.
Which model do you prefer? Why?

Mid-Unit Review

LESSON

- 9.1 1.** Draw a number line. Show each integer on the number line:
 $+3, -2, -5, 0$
- 2.** Use an integer to represent each situation.
- a) 12°C below zero
 - b) a golf score of 3 strokes above par
 - c) 10 m above sea level
 - d) a drop of \$2 in the price of a stock
 - e) \$25 earned
 - f) a mountain elevation of 1500 m
- 9.2 3.** Order these integers from least to greatest.
- a) $+4, -3, -2, +1, -4$
 - b) $+18, +50, 0, -50, -17$
- 9.3 4.** Use red and yellow tiles to model each integer two different ways. Draw the tiles.
- a) -5 b) 0
 - c) $+8$ d) -1
 - e) $+3$ f) -7
- 9.4 5.** Use coloured tiles to add. Draw pictures of the tiles you used.
- a) $(+4) + (-1)$
 - b) $(-3) + (-2)$
 - c) $(-5) + (+1)$
 - d) $(+6) + (+3)$
 - e) $(-4) + (-8)$
 - f) $(+4) + (+8)$
- 9.5 6.** Use a number line to add.
- a) $(+3) + (+2)$ b) $(-5) + (-1)$
 - c) $(-10) + (+8)$ d) $(+6) + (-5)$
 - e) $(-8) + (+8)$ f) $(-5) + (+12)$
- 7.** Use patterns to add.
- a) $(+6) + (-3)$ b) $(-2) + (+8)$
 - c) $(+5) + (-9)$ d) $(-4) + (+9)$
- 8.** Add.
- a) $(+4) + (+1) + (-2)$
 - b) $(-3) + (-1) + (-4)$
 - c) $(-5) + (+1) + (+3)$
 - d) $(+6) + (-5) + (-8)$
- 9.** a) Puja earned \$50, was given \$10, and spent \$20.
How much did Puja then have?



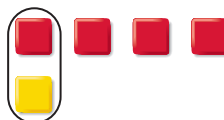
- b) The temperature starts at $+5^{\circ}\text{C}$, goes up 2°C , then drops 10°C . What is the final temperature?
 - c) The population of a city was 124 000, then it dropped by 4000 people. What was the population then?
- 10.** a) Add. $(+4) + (-5)$
b) Find another pair of integers that has the same sum as part a. Do this 3 times.

To add integers, we combine groups of tiles.
To subtract integers, we do the reverse:
we “take away” tiles from a group.

Recall that equal numbers of
red and yellow tiles model 0.
For example, $(-5) + (+5) = 0$



Adding 0 to a set of tiles does
not change its value.
For example, $(-3) + 0 = -3$



Explore

Work with a partner.
You will need coloured tiles.
Use tiles to subtract. Sketch the tiles you used in each case.

- $(+5) - (+3)$
- $(+5) - (-3)$
- $(-3) - (+5)$
- $(-3) - (-5)$

Reflect & Share

Compare your results with those of another pair of classmates.
Did they draw the same sets of tiles? Explain.
When you subtracted, how did you know how many tiles to use
to model each integer?

Connect

To use tiles to subtract integers, we model the first integer, then take
away the number of tiles indicated by the second integer.

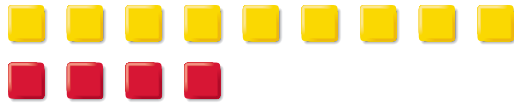
We can use tiles to subtract: $(+5) - (+9)$

Model +5.

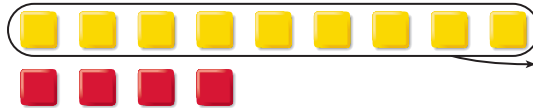


There are not enough tiles to take away $+9$.
To take away $+9$, we need 4 more yellow tiles.

We add zero pairs without changing the value.
Add 4 yellow tiles and 4 red tiles. They represent 0.



By adding 0, the integer the tiles represent has not changed.
Now take away the 9 yellow tiles.



Since 4 red tiles remain, we write: $(+5) - (+9) = -4$

Example

Use tiles to subtract.

a) $(-2) - (-6)$ b) $(-6) - (+2)$ c) $(+2) - (-6)$

Solution

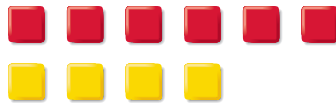
a) $(-2) - (-6)$

Model -2 .

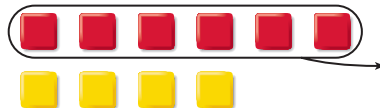


There are not enough tiles to take away -6 .
To take away -6 , we need 4 more red tiles.

We add zero pairs without changing the value.
Add 4 red tiles and 4 yellow tiles.



Now take away 6 red tiles.



Since 4 yellow tiles remain, we write: $(-2) - (-6) = +4$

b) $(-6) - (+2)$

Model -6 .

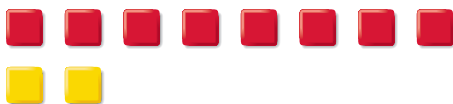


There are no yellow tiles to take.

We need 2 yellow tiles to take away.

We add zero pairs.

Add 2 yellow tiles and 2 red tiles.



Now take away 2 yellow tiles.



Since 8 red tiles remain, we write: $(-6) - (+2) = -8$

c) $(+2) - (-6)$

Model $+2$.

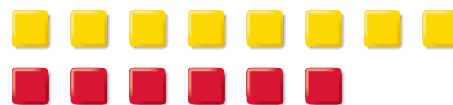


There are no red tiles to take.

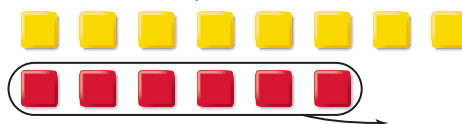
We need 6 red tiles to take away.

We add zero pairs.

Add 6 red tiles and 6 yellow tiles.



Now take away 6 red tiles.



Since 8 yellow tiles remain, we write: $(+2) - (-6) = +8$

Notice the results in the *Example*, parts b and c.

When we reverse the order in which we subtract two integers, the answer is the opposite integer.

$$(-6) - (+2) = -8$$

$$(+2) - (-6) = +8$$

Practice

- Use tiles to subtract. Draw pictures of the tiles you used.
 - $(+7) - (+4)$
 - $(-2) - (-2)$
 - $(-9) - (-6)$
 - $(+4) - (+2)$
 - $(-8) - (-1)$
 - $(+3) - (+3)$
- Use tiles to subtract.
 - $(-1) - (-4)$
 - $(+3) - (+8)$
 - $(-4) - (-11)$
 - $(+7) - (+8)$
 - $(-4) - (-6)$
 - $(+1) - (+10)$
- Subtract.
 - $(-4) - (-1)$
 - $(+8) - (+3)$
 - $(-11) - (-4)$
 - $(+8) - (+7)$
 - $(-6) - (-4)$
 - $(+10) - (+1)$
- Subtract.
 - $(+4) - (-7)$
 - $(-2) - (+8)$
 - $(-9) - (+5)$
 - $(+6) - (-8)$
 - $(-3) - (+6)$
 - $(-5) - (+7)$
- Subtract.
 - $(+4) - (+5)$
 - $(-3) - (+5)$
 - $(-4) - (+3)$
 - $(-1) - (-8)$
 - $(+8) - (-2)$
 - $(+4) - (-7)$
- Write 3 integer subtraction problems.
Trade problems with a classmate.
Solve your classmate's problems.
 - Use coloured tiles to subtract each pair of integers.
 - $(+3) - (+1)$ and $(+1) - (+3)$
 - $(-3) - (-2)$ and $(-2) - (-3)$
 - What do you notice about each pair of problems in part a?
- $(+5) - (-2) = +7$
Predict the value of $(-2) - (+5)$.
Explain the reasoning for your choice.
- Assessment Focus** Use integers.
Write a subtraction problem that would give each answer.
How many problems can you write each time?
 - $+2$
 - -3
 - $+5$
 - -6



Calculator Skills

Assume the \div key on your calculator does not work.

Find two ways to use your calculator to divide: $888 \div 24$

10. Here is a magic square:

0	+5	-2
-1	+1	+3
+4	-3	+2

- a) Subtract +4 from each entry.
Is it still a magic square? Why?
- b) Subtract -1 from each entry.
Is it still a magic square? Why?
- c) Make up your own magic square.

Take It Further

11. a) Find two integers with a sum of -1 and a difference of +5.
b) Create and solve a similar integer problem.

12. Evaluate.

- a) $(+4) + (+1) - (+3)$ b) $(+1) - (+2) - (-1)$
c) $(-3) - (+1) + (+4)$ d) $(-2) - (-4) + (-1)$
e) $(+2) - (+1) - (+4)$ f) $(+1) - (+2) + (+1)$

13. Copy and complete.

- a) $(+4) - \square = +3$
b) $(+3) - \square = -1$
c) $\square - (+1) = +4$

14. Which expression in each pair has the greater value?

Explain your reasoning.

- a) i) $(+3) - (-1)$ ii) $(-3) - (+1)$
b) i) $(-4) - (-5)$ ii) $(+4) - (+5)$

Reflect

Here are 4 types of subtraction problems:

- (negative integer) - (negative integer)
- (negative integer) - (positive integer)
- (positive integer) - (positive integer)
- (positive integer) - (negative integer)

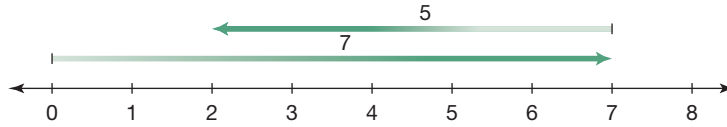
Write a problem for each type of subtraction.

Show how you use tiles to solve each problem.

Focus Subtract integers using number lines and patterns.

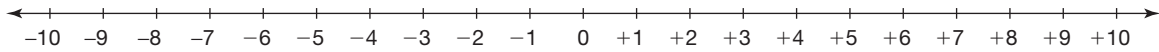
Just as we can show addition of whole numbers on a number line, we can show subtraction of whole numbers on a number line.

$$7 - 5 = 2$$



Explore

Work with a partner.
You will need copies of this number line.



- Use the number line to subtract.

$(+6) - (-4)$	$(-3) - (+5)$
$(-5) - (-4)$	$(+7) - (+10)$

Check your answers using coloured tiles.



- Subtract. Add.

$(+7) - (+2)$	$(+7) + (-2)$
$(-7) - (-2)$	$(-7) + (+2)$
$(+7) - (-2)$	$(+7) + (+2)$
$(-7) - (+2)$	$(-7) + (-2)$

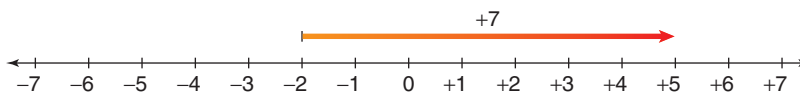
What do you notice about the answers in each row?
What patterns do you see in each subtraction and addition?
Check your pattern using other integers.

Reflect & Share

Compare your answers with those of another pair of classmates.
How can you use addition to subtract two integers?

- To subtract two whole numbers, such as $5 - 2$, we think, “What do we add to 2 to get 5?”
We add 3 to 2 to get 5; so, $5 - 2 = 3$

We can do the same to subtract two integers.
For example, to subtract: $(+5) - (-2)$
Think: “What do we add to -2 to get $+5$?”



We add $+7$ to -2 to get $+5$; so, $(+5) - (-2) = +7$
We also know that $(+5) + (+2) = +7$.
We can look at other subtraction equations and related addition equations.

$(+9) - (+4) = +5$	$(+9) + (-4) = +5$
$(-9) - (-4) = -5$	$(-9) + (+4) = -5$
$(-9) - (+4) = -13$	$(-9) + (-4) = -13$
$(+9) - (-4) = +13$	$(+9) + (+4) = +13$



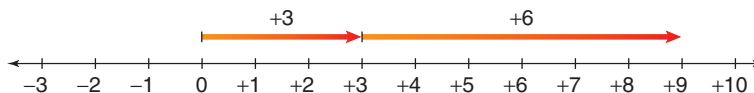
In each case, the result of subtracting an integer is the same as adding the opposite integer.

For example,

$(-9) - (+4) = -13$	$(-9) + (-4) = -13$
$\underbrace{\hspace{2em}}$ \downarrow Subtract +4.	$\underbrace{\hspace{2em}}$ \downarrow Add -4.

- To subtract an integer, we add the opposite integer.
For example, to subtract: $(+3) - (-6)$
Add the opposite: $(+3) + (+6)$

The opposite of -6 is $+6$.



So, $(+3) - (-6) = +9$

Example 1

Use a number line to subtract.

a) $(+2) - (+9)$

b) $(+2) - (-9)$

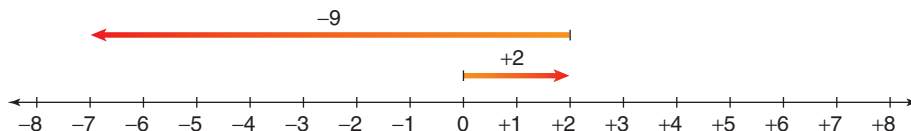
Solution

a) To subtract: $(+2) - (+9)$

Add the opposite: $(+2) + (-9)$

Use a number line.

$$(+2) + (-9) = -7$$

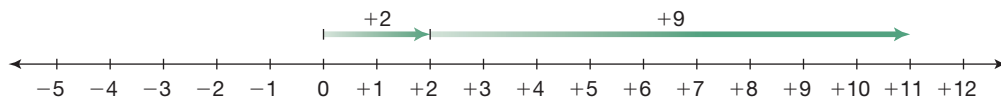


b) To subtract: $(+2) - (-9)$

Add the opposite: $(+2) + (+9)$

Use a number line.

$$(+2) + (+9) = +11$$



Example 2

On April 27, 2004, the highest temperature in Windsor was $+5^{\circ}\text{C}$ and the highest temperature in Yellowknife was -2°C .

What is the difference in temperature? Interpret the answer.

Solution

The difference between $+5$ and -2 can be written in two ways:

$$(+5) - (-2)$$

$$(-2) - (+5)$$

Add the opposite: $(+5) + (+2)$

Add the opposite: $(-2) + (-5)$

$$(+5) + (+2) = +7$$

$$(-2) + (-5) = -7$$

The difference in temperature is $+7^{\circ}\text{C}$.

The difference in temperature is -7°C .

The temperature in Windsor is 7°C greater than the temperature in Yellowknife.

The temperature in Yellowknife is 7°C less than the temperature in Windsor.

Practice

- Rewrite using addition.
 - $(+6) - (+4)$
 - $(-5) - (+4)$
 - $(-2) - (-3)$
 - $(+4) - (-2)$
 - $(+1) - (+1)$
 - $(+1) - (-1)$
- Use a number line to subtract.
Use coloured tiles to check your answers.
 - $(+2) - (+1)$
 - $(+4) - (-3)$
 - $(-4) - (-1)$
 - $(-5) - (+2)$
 - $(-2) - (-6)$
 - $(-3) - (-7)$
- Reverse the order of the integers in question 2, then subtract.
 - How are the answers different from those in question 2? Explain.
- Use a number line to subtract.
 - $(+10) - (+5)$
 - $(+7) - (-3)$
 - $(-8) - (+6)$
 - $(-10) - (+5)$
 - $(-4) - (+4)$
 - $(-4) - (-4)$
- Find the difference between:
 - a temperature of $+3^{\circ}\text{C}$ and -5°C
 - a temperature of -15°C and -10°C
 - an elevation of 5 m above sea level and one of 2 m below sea level
 - a golf score of 1 below par and one of 3 below par
- The table shows the average afternoon temperatures in January and April for four Canadian cities. Find the difference between the temperatures in April and January for each city. Show your work.



	City	January Temperature	April Temperature
a)	Calgary	-4°C	$+13^{\circ}\text{C}$
b)	Iqaluit	-22°C	-10°C
c)	Toronto	-3°C	$+12^{\circ}\text{C}$
d)	Victoria	$+7^{\circ}\text{C}$	$+13^{\circ}\text{C}$

Calculator Skills

Find each answer.

- $15 \times (32 + 41)$
- $198 - 104 \div 13$
- $14 \times 15 - 16 \times 9$

7. Use patterns to subtract.

- a) Subtract: $(+2) - (+5)$
Start the pattern with $(+6) - (+5) = +1$.
- b) Subtract: $(+7) - (-3)$
Start the pattern with $(+7) - (+4) = +3$.
- c) Subtract: $(-3) - (+7)$
Start the pattern with $(+8) - (+7) = +1$.

8. Assessment Focus

- a) Subtract: $(-6) - (+11)$
- b) Suppose we subtract the integers in the opposite order:
 $(+11) - (-6)$
How does the answer compare with the answer in part a?
Use number lines or patterns to explain.
- c) How is $(+6) - (-11)$ different from $(-6) - (+11)$?
Explain.

9. This table shows the mean daily maximum temperatures in degrees Celsius for Rankin Inlet for each month.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
-28	-28	-23	-12	-2	+8	+16	+14	+6	-3	-16	-24

- a) What is the median?
- b) What is the range?
- c) What is the mode?

10. Evaluate.

- a) $(+4) - (+2) - (+1)$ b) $(-2) - (+1) - (-4)$
- c) $(-1) + (-2) - (+1)$ d) $(+5) - (+1) + (-2)$
- e) $(+10) - (+3) - (-5)$ f) $(-7) - (+1) + (-3)$

11. Show three ways that $+4$ can be written as the difference of two integers.

Reflect

Choose two integers: one positive, one negative.

Add the integers. Subtract the integers.

Explain how the subtraction of two integers is related to addition.



Choosing a Strategy

Strategies

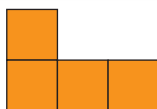
- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and check.
- Make an organized list.
- Use a pattern.
- Draw a graph.
- Use logical reasoning.



1. A textbook is opened at random. The product of the page numbers is 23 256. What is the number on the left hand page?
2. Ms. Pantuso has a bag of candies that she wants to share equally among the students in her class.
On Monday, all 20 students were present. If she had shared the candies, she would have had 7 candies left over.
On Tuesday, two boys were absent. If she had shared the candies, she would again have had 7 left over.
On Wednesday, the two boys were still absent and one girl was absent. Ms. Pantuso shared all the candies among the students with none left over.
 - a) What was the least number of candies Ms. Pantuso could have had?
 - b) How many candies did each student receive?
3. Briony wants to print copies of her new brochure. The local print shop charges 15¢ a copy for the first 25 copies, 12¢ a copy for the next 50 copies, and 8¢ a copy for any additional copies.
How much would Briony pay for each number of copies?
 - a) 60 copies
 - b) 240 copies
4. What is the greatest number of 5 cm by 3 cm rectangles that can be cut from a 20 cm by 10 cm sheet of cardboard?
5. Mr. Anders estimates it will take him 4 h to build a wall. His assistant would take 6 h to build the wall.
 - a) What fraction of the wall could each person build in one hour?
 - b) Suppose they worked together. What fraction could they build in one hour?
 - c) How long would it take them to build the wall?
6. The mean of five numbers is 13.
The mode is 16.
The median is 14.
Write two sets of numbers that satisfy these conditions.

7. A rectangular prism has a square base, and a volume of 36 cm^3 .
What are the possible whole number dimensions of the prism?

8. Copy this figure on grid paper.
Add one square to make the figure symmetrical. How many solutions can you find?



9. Eight students wrote a math test. Their mean mark was 54%.
Another student wrote the test and scored 99%.
What is the mean mark for the nine students?

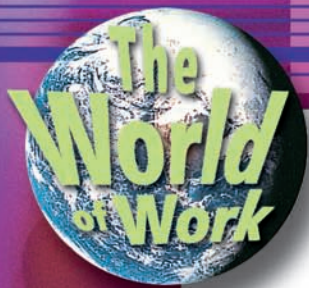
10. Write 45 as the sum of consecutive integers.
How many solutions can you find?

11. a) The product of two whole numbers is 1000.
No number contains a zero.
What are the numbers?
How many different answers can you find?

b) The product of three whole numbers is 1000.
No number contains a zero.
What are the numbers?
How many different answers can you find?

12. A book contains 124 pages numbered from 1 to 124.
How many times does the digit 7 appear?





Historian

The modern historian does a lot of ‘detective’ work, especially when reading old documents or journals that describe timelines. Adding or subtracting days, months, or years from a known date is a regular part of the work. The historian must deal with modern timekeeping conventions, such as Common Era (CE) and Before Common Era (BCE). But he must also know that, throughout history, different societies had different ways of describing time. Often it was simply recorded as a period of time before or after another significant date or event.

Suppose you read this entry in an 18th century explorer’s journal dated May 21:

**A fortnight is
2 weeks.**

“We have been following this river for a fortnight. Three days into our journey we came upon a great waterfall blocking our path and had to portage around it. But the river has been calm since.”

How would you be able to calculate what date the waterfall was discovered?



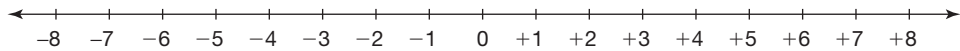
Suppose you knew that Augustus Caesar died in 14 CE at the age of 77. Can you calculate the year of his birth? Explain.



What Do I Need to Know?

✓ Integer Models

- You can use a number line to show integers.



Negative numbers are to the left of 0. Positive numbers are to the right of 0.

- Coloured tiles model integers.

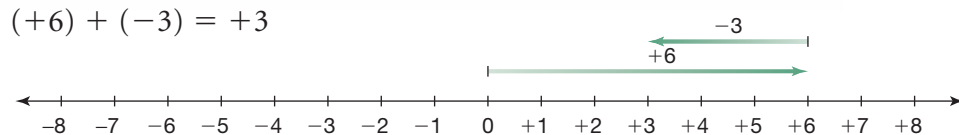


✓ Adding Integers

- You can use tiles to add integers.



- You can use a number line to add integers.



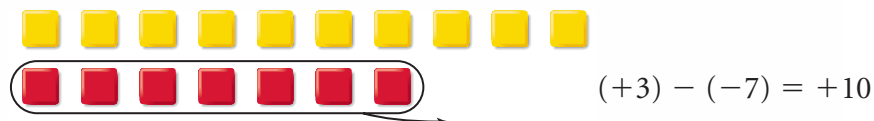
✓ Subtracting Integers

- You can use tiles to subtract integers: $(+3) - (-7)$

We need enough red tiles to take away 7 of them.

Model +3:

Since there are not enough tiles to take away -7 , add 7 yellow tiles and 7 red tiles. Now take away 7 red tiles. There are 10 yellow tiles left.



- You can also subtract by adding the opposite:

$$\begin{aligned} (-5) - (-8) &= -5 + (+8) \\ &= +3 \end{aligned}$$

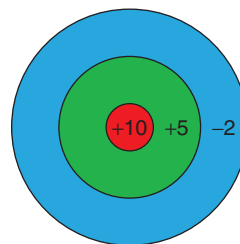
LESSON

- 9.1** 1. Draw a number line. Mark each integer on the number line:
+3, -5, +1, -2
2. Use an integer to represent each situation.
- a golf score of 2 strokes under par
 - 250 m below sea level
 - 32°C
 - a loss of \$125
 - an increase of \$3 in the monthly cost of cable television
- 9.2** 3. Order these integers from least to greatest:
+200, -55, +150, -3, -54
- 9.4** 4. Use tiles to add or subtract.
- 9.6** a) $(-1) + (+3)$
b) $(+3) + (-4)$
c) $(-2) - (+3)$
d) $(-1) - (-3)$
- 9.5** 5. Use a number line to add or subtract.
- 9.7** a) $(-1) + (+3)$
b) $(+6) + (-4)$
c) $(-4) - (+6)$
d) $(-5) - (-3)$
6. When you add two positive integers, their sum is always a positive integer. When you subtract two positive integers, is their difference always a positive integer? Explain.
7. At midnight in North Bay, the temperature was -5°C . During the next 24 h, the temperature rose 12°C , then dropped 9°C . What was the final temperature? Show your work.
- 9.6** 8. Use tiles or a number line to subtract.
- 9.7** a) $(+4) - (+1)$
b) $(+5) - (-1)$
c) $(+2) - (-2)$
d) $(-4) - (+1)$
e) $(-6) - (-2)$
f) $(-10) - (-5)$
g) $(-4) - (-2)$
9. Use tiles or a number line. Find the difference between:
- a temperature of $+5^{\circ}\text{C}$ and -7°C
 - an elevation of -100 m and $+50$ m
 - a golf score of 1 over par and 2 under par
10. Subtract.
- $(+3) - (+1)$
 - $(-5) - (-2)$
 - $(+100) - (+60)$
 - $(-100) - (+60)$
11. a) Find 5 pairs of integers with a sum of -6 .
b) Find 5 pairs of integers with a difference of -3 .

Practice Test

- We measure time in hours.
Suppose 12 noon is represented by the integer 0.
 - Which integer represents 1 p.m. the same day?
 - Which integer represents 10 a.m. the same day?
 - Which integer represents 12 midnight the same day?
 - Which integer represents 10 p.m. the previous day?
- Order all the integers in question 1 from least to greatest.
- Evaluate.
 - $(+5) + (-8)$
 - $(-3) - (+7)$
 - $(-9) + (-1)$
 - $(-4) + (+10)$
 - $(-6) - (-2)$
 - $(+12) - (-11)$
- Without calculating the sum, how can you tell if the sum of two integers will be:
 - zero?
 - negative?
 - positive?Include examples in your explanations.
- Here is a different type of dartboard.

A player throws 3 darts at the board.
His score is the sum of the integers in the areas his darts land.
Assume all 3 darts hit the board.
 - How many different scores are possible?
 - Find each score.
- The lowest temperature possible is approximately -273°C .
The temperature at which water boils is 100°C .
What is the difference in these temperatures?



TIME ZONES



The map shows the world's time zones. Greenwich, in London, England, is the reference point, or the zero for the time zones.

Its time is called UTC, or Coordinated Universal Time.

London, England, is also in this time zone.

The positive and negative integers on the map show the changes in time from UTC.

The 2008 Summer Olympics will be held in Beijing, China.

1. The local start times of some Olympic events are given. If family members want to watch these events live, in Sudbury (the same time zone as Toronto), what time should they “tune in”?
 - a) 200-m backstroke at 2:00 p.m.
 - b) 100-m dash at 7:00 p.m.
 - c) gymnastics at 11:00 p.m.
 - d) middleweight boxing at 8:00 a.m.

2. An event is broadcast live in Montreal at 9:00 p.m.
What time is it taking place in Beijing?
3. Two pen pals plan to meet in Beijing for the Olympics.
Atsuko lives in Tokyo, Japan.
She can get a direct flight to Beijing that takes 4 h.
Paula lives in Sydney, Australia, and her direct flight takes 13 h.
What time does each girl leave her country to arrive in Beijing at 6 p.m., Beijing time?

Check List

Your work should show:

- ✓ how you used integers to solve each problem
- ✓ the strategies you used to add and subtract integers
- ✓ correct calculations
- ✓ a clear explanation of how integers relate to time zones



4. Olympic funding depends on money from North American television networks. What problems will the organizers of the Beijing Olympics encounter when they plan the times for events?
5. Make up your own problem about the time zone map.
Solve your problem.

Show your work. Show how you can use integers to solve each problem.

Reflect on the Unit

Choose a positive integer and a negative integer.

How many different ways can you add them? Subtract them?

Show your work.

If you used number lines, patterns, or coloured tiles, draw pictures to show how you used them.