UNIT 7 Geometry

Patterns are pleasing to the eye. They are used by designers, architects, and engineers to make their products more attractive.
Look at the quilt pattern.
Which figures are used as quilt blocks?
Which other figures have you seen in quilts?

What You'll Learn

- Identify, describe, compare, and classify figures.
- Identify the conditions that make two figures congruent.
- Construct and analyse tiling patterns.
- Recognize the image of a figure after a transformation.
- Create and analyse designs using transformations.

Why It's Important

- Geometry is used daily by scientists, architects, engineers, and land developers.
- Geometric attributes, such as congruence and symmetry, enable you to see the world around you in a different way.

Key Words

- convex polygon
- concave polygon
- tiling the plane
- tessellations

UNIT 7

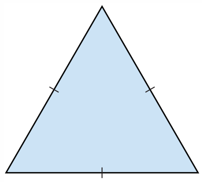
Skills You'll Need

Classifying Triangles

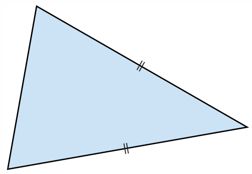
Here are two ways to classify triangles.

- By side length

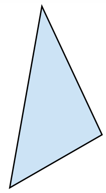
An equilateral triangle has all sides equal.



An isosceles triangle has 2 sides equal.

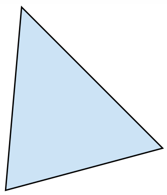


A scalene triangle has all sides different.

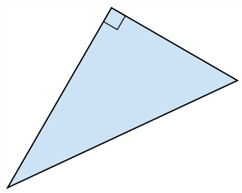


- By angle measure

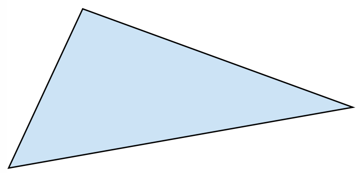
An acute triangle has all angles less than 90°.



A right triangle has one 90° angle.



An obtuse triangle has one angle greater than 90°.



✓ Check

Use square dot paper or isometric dot paper.

1. Draw an isosceles triangle.
Is it acute, obtuse, or right? How do you know?

2. Draw an obtuse triangle.
Is it equilateral, scalene, or isosceles? How do you know?

3. Can you draw an obtuse isosceles triangle?
If you can, draw it.
If you cannot draw the triangle, say why it cannot be drawn.

4. Can you draw a right equilateral triangle?
If you can, draw it.
If you cannot draw the triangle, say why it cannot be drawn.

Constructing a Triangle

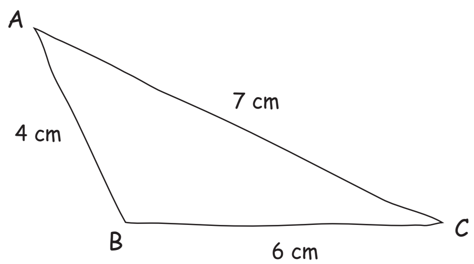
Here are two ways to construct a triangle, using a ruler, compass, and protractor.

Example 1

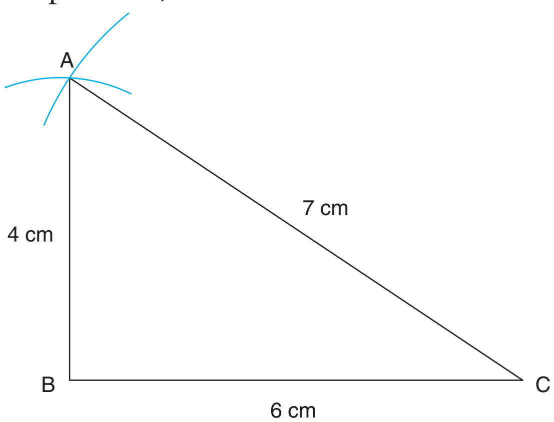
Construct △ABC with AB = 4 cm, BC = 6 cm, and CA = 7 cm.

Solution

You will need a ruler and compass.
Step 1 Sketch the triangle.



Step 2 Construct the triangle:
Use a ruler to draw side BC = 6 cm.
With the compass point and pencil 7 cm apart,
put the compass point on C and draw an arc.
All points on this arc are 7 cm from C.
With the compass point and pencil 4 cm apart,
put the compass point on B and draw an arc.
All points on this arc are 4 cm from B.
Make sure the arc intersects the first arc you drew.
Mark a point where the arcs intersect.
This point is 7 cm from C and 4 cm from B.
Label the point A. Join AB and AC. Label each side with its length.



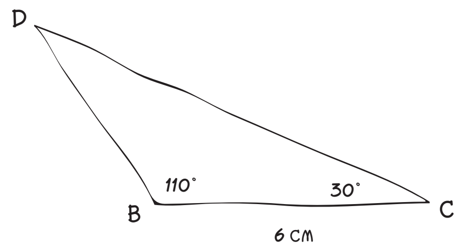
Example 2

Construct $\triangle BCD$ with $BC = 6$ cm, $\angle B = 110°$, and $\angle C = 30°$.

Solution

You will need a ruler and protractor.

Step 1 Sketch the triangle.



Step 2 Construct the triangle:

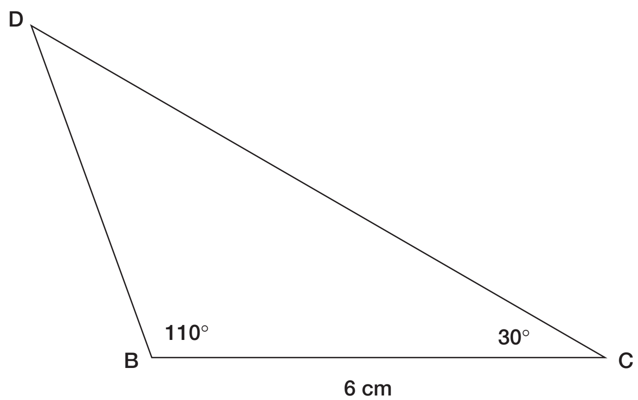
Use a ruler to draw side $BC = 6$ cm.

Use a protractor to make an angle of 110° at B.

Use a protractor to make an angle of 30° at C.

Label point D where the arms of the angles intersect.

Label the known side and angles.



✓ Check

5. Construct each triangle.

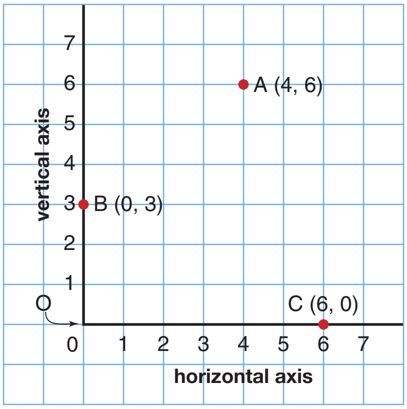
a) $\triangle CDE$ with $CD = 4$ cm, $DE = 7$ cm, $CE = 9$ cm

b) $\triangle DEF$ with $DE = 7$ cm, $\angle D = 80°$, $\angle E = 30°$

Plotting Points on a Coordinate Grid

When we draw a horizontal axis and a vertical axis on grid paper, we have a coordinate grid.
The axes intersect at the origin, O.

We label each axis with numbers, beginning with 0 at the origin.



A point on a grid is described by its coordinates.

Point A has coordinates (4, 6).
To plot point A, start at 4 on the horizontal axis, then move up 6 spaces.
Mark a point. This is point A.

Point B has coordinates (0, 3).
To plot point B, start at 0, then move up 3 spaces. Point B is on the vertical axis.

Point C has coordinates (6, 0).
To plot point C, mark a point at 6 on the horizontal axis.

✓ Check

6. On grid paper, draw a coordinate grid.
Plot each point on the grid.
A(5, 7), B(3, 8), C(10, 4), D(9, 1), E(0, 8), F(5, 0)

7. **a)** Where are all the points with horizontal coordinate 0?
b) Where are all the points with vertical coordinate 0?

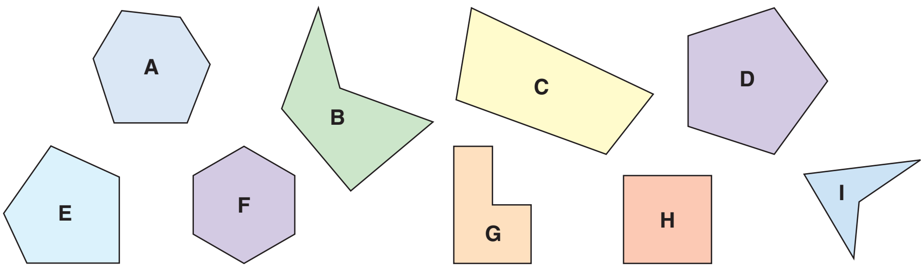
7.1 Classifying Figures

Focus Identify, describe, and classify geometric figures.

Look around the classroom.
Name the different figures you see.
Which figure is most common?

Explore

Work with a group.
You will need a ruler and a protractor.
Your teacher will give you a large copy of these figures.



- ➢ Identify each figure.
  Describe it.
- ➢ Choose two figures.
  How many different ways can you compare them?
- ➢ Choose three figures. How are they the same?
  How are they different?

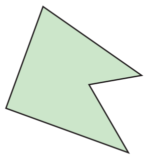
Reflect & Share

Share your results with another group of classmates.
Work together to classify the figures in different ways.

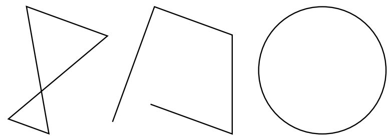
Connect

A polygon is a closed figure with sides that are line segments.
Exactly 2 sides meet at a vertex.
The sides intersect only at the vertices.

This figure is a polygon.



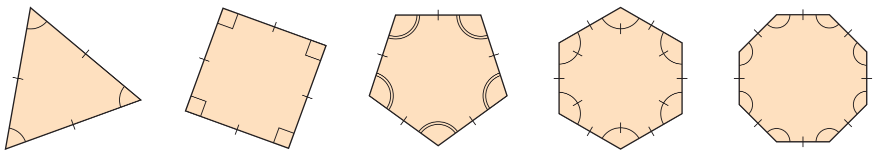
These figures are *not* polygons.



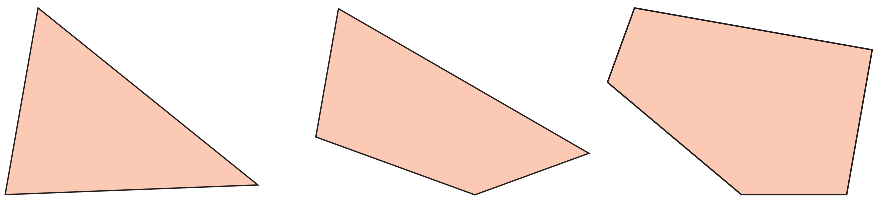
A regular polygon has all sides equal and all angles equal.
These polygons are regular.

A regular polygon has line symmetry and rotational symmetry.

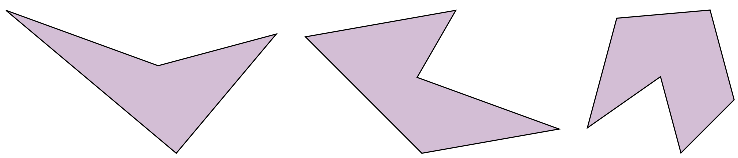
Recall that matching arcs or symbols in angles show that the angles are equal.



A **convex polygon** has all angles less than 180°.
These polygons are convex.



A **concave polygon** has at least one angle greater than 180°.
These polygons are concave.

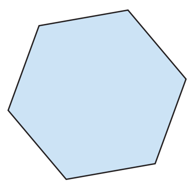


Example

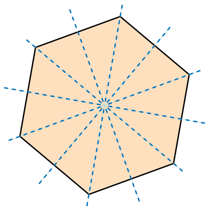
Here is a regular hexagon.

a) How many lines of symmetry does it have?

b) What is the rotational symmetry?



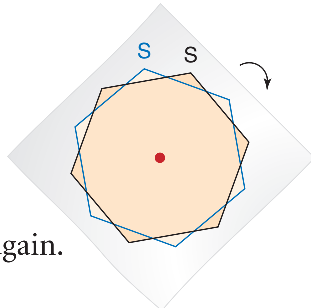
Solution



a) Trace the hexagon.
Fold the tracing paper so that one part of the hexagon coincides with the other.
The fold line is a line of symmetry.
Repeat the folding as many times as possible.

A regular hexagon has 6 lines of symmetry:
3 lines join opposite vertices, and 3 lines join the midpoints of opposite sides.

b) Trace the hexagon. Place the tracing to coincide with the hexagon.
Rotate the tracing about its centre until the tracing coincides with the hexagon again.
Count how many times you can do this.



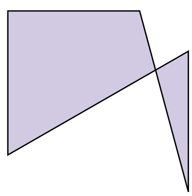
Label two corresponding vertices. Then you know when the tracing is back at the starting position.

The tracing coincides with the hexagon 6 times.
So, a regular hexagon has rotational symmetry of order 6.

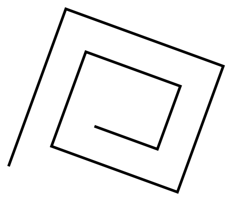
Practice

1. Explain why each figure is not a polygon.

a)

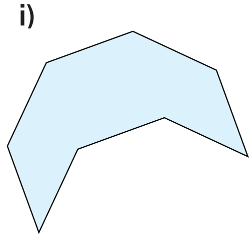


b)

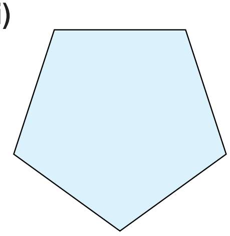


2. **a)** Is each polygon regular? How do you know?

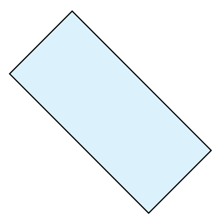
i)



ii)

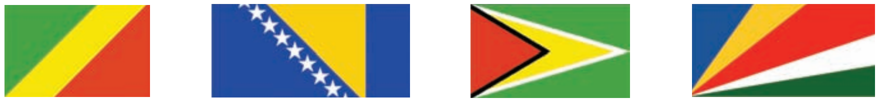


iii)

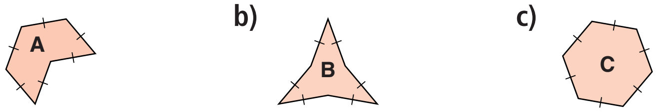


b) Which polygons in part a have:
i) line symmetry? **ii)** rotational symmetry?
How do you know?

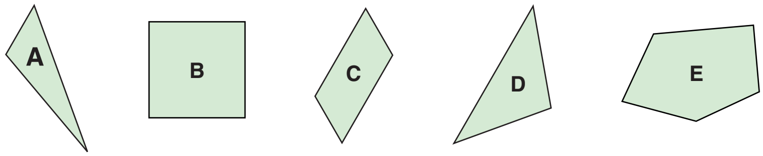
3. Identify the figures in each flag.
Describe each figure as many ways as you can.
a) Congo **b)** Bosnia-Herzegovina **c)** Guyana **d)** Seychelles



4. Describe each figure. How are the figures the same? Different?
a) **b)** **c)**



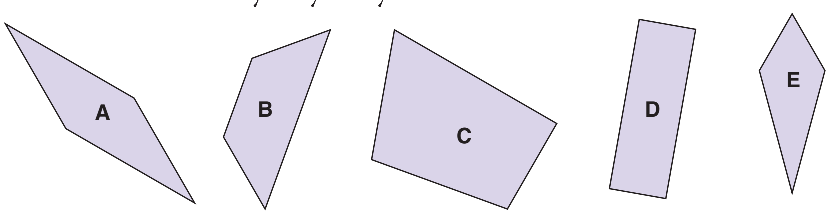
5. Match each polygon with its description below.



a) an isosceles triangle with an angle of 40°
b) a rhombus with a right angle
c) a pentagon with an angle of 120°
d) a parallelogram with an angle of 60°
e) an obtuse triangle with an angle of 110°

6. Use square dot paper or isometric dot paper.
Draw each polygon.
a) an isosceles triangle with a height of 4 units
b) a parallelogram with an angle of 45°
c) a trapezoid with a 90° angle and a 45° angle
d) a kite with exactly one right angle
e) a parallelogram with a 90° angle
f) a scalene obtuse triangle
g) an isosceles right triangle
h) a hexagon with exactly 3 right angles

7. Identify each polygon.
Describe it as many ways as you can.

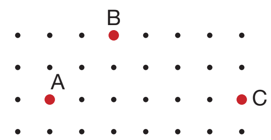


A quadrilateral is a polygon with 4 sides.

8. Use dot paper.
- **a)** Draw a quadrilateral. Label it A.
- **b)** Draw another quadrilateral that differs from quadrilateral A in only one way. Label it B.
- **c)** Continue to draw quadrilaterals that differ in only one way. Label each one you draw. How many different quadrilaterals can you draw?

9. Assessment Focus

The 3 points A, B, C are vertices of a polygon. Copy the points on dot paper.
- **a)** Find other vertices and sketch each figure.
  - **i)** a trapezoid with line symmetry
  - **ii)** a kite
  - **iii)** a parallelogram
  - **iv)** a pentagon



- **b)** How many other figures can you make that have these points as 3 vertices? Identify each figure. Describe it as many ways as you can.

10. The lengths of three sides of a quadrilateral are 5 cm, 5 cm, and 8 cm.
- **a)** Sketch and name the different quadrilaterals possible.
- **b)** Suppose one angle is 90°. Which quadrilaterals are possible now? Justify your answer.

Number Strategies

Find the next 3 numbers in each pattern.

What is each pattern rule?

- 23, 28, 26, 31, 29,…
- 6, 9, 15, 27, 51,…
- 1, 3, 9, 27,…

Math Link

Your World

The Department of Highways uses different figures for road signs. Which road signs use each of these figures: pentagon, octagon, square, circle, rectangle, triangle?

Reflect

Choose 3 different polygons. Sketch each polygon as many different ways as you can. Describe each polygon.

7.2 Congruent Figures

Focus Identify the conditions for congruence.

Explore

Work on your own.
You will need a ruler, protractor, and compass.
For each set of measurements given, how many different triangles can you draw?

➢ Construct a triangle with sides of length 5 cm, 7 cm, and 9 cm.
➢ Construct a triangle with two sides of length 9 cm and 5 cm, and one angle of 30°.
➢ Construct a triangle with one side of length 5 cm and two angles of 40° and 60°.

Reflect & Share

Compare your triangles with those of several classmates.

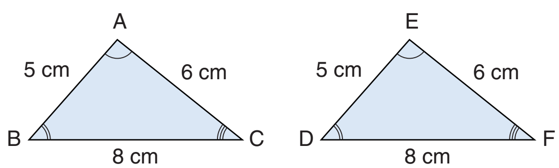
➢ How many different triangles can you draw in each case?
- when you know 3 sides
- when you know 2 sides and 1 angle
- when you know 1 side and 2 angles

➢ What measurements do you need to know to be able to draw exactly one triangle?

Connect

➢ When 3 sides of a triangle are given, only one triangle can be drawn. So, if we know that two triangles have the same 3 sides, those triangles must be congruent.

Congruent figures have the same size and shape. These triangles are drawn to scale.



We say: Triangle ABC is congruent to triangle EDF.
We write: $\triangle ABC \cong \triangle EDF$

We list the corresponding vertices of the triangles in the same order.

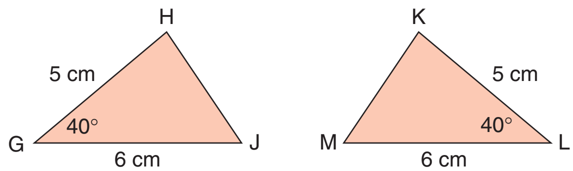
△ABC and △EDF have:

corresponding sides equal	and	corresponding angles equal
AB = ED		∠A = ∠E
BC = DF		∠B = ∠D
AC = EF		∠C = ∠F

➢ When 2 sides and 1 angle of a triangle are given, there are two cases to consider.

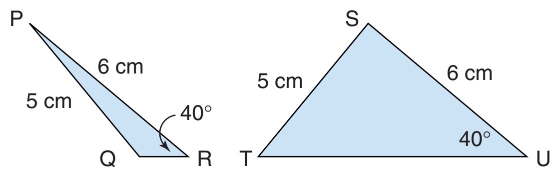
- The given angle is between the 2 sides.
  Only one triangle can be drawn.

You may have to flip or rotate one triangle so both triangles face the same way.



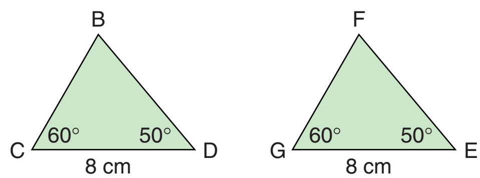
So, triangle GHJ is congruent to triangle LKM or
△GHJ ≅ △LKM

- The given angle is *not* between the 2 sides.
  Sometimes more than one triangle can be drawn.



△PQR and △STU are *not* congruent.

➢ When 2 angles and 1 side are given, only one triangle can be drawn.

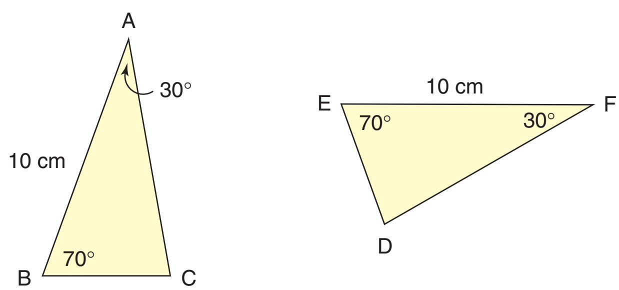


△BCD ≅ △FGE

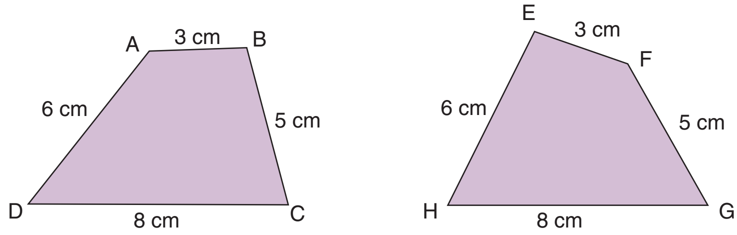
Example

Are the figures in each pair congruent? How do you know?

a)



b)



Solution

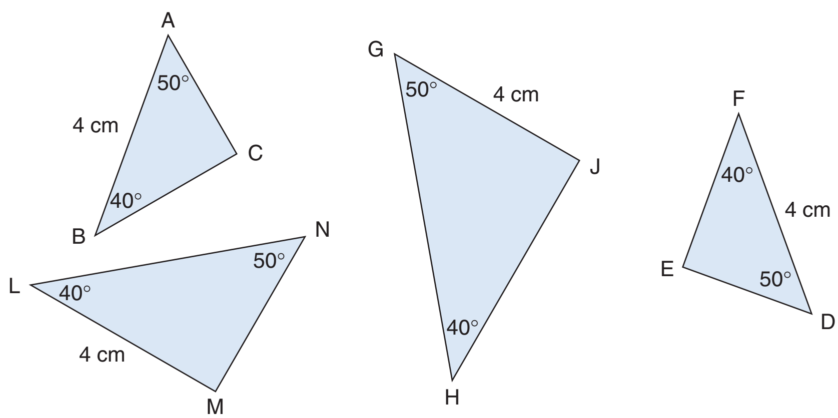
a) △ABC and △FED have 2 pairs of corresponding angles equal and 1 pair of corresponding sides equal:
∠A = ∠F = 30°
∠B = ∠E = 70°
AB = FE = 10 cm
So, △ABC and △FED are congruent: △ABC ≅ △FED

b) Quadrilateral ABCD and quadrilateral EFGH have 4 pairs of corresponding sides equal. But the quadrilaterals have different shapes. So, the quadrilaterals are not congruent.

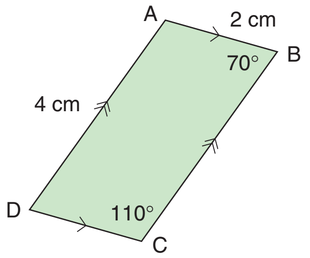
Part b of the *Example* shows that for two quadrilaterals to be congruent, it is not sufficient that 4 pairs of corresponding sides are equal. We need to know that the corresponding angles are equal, too.

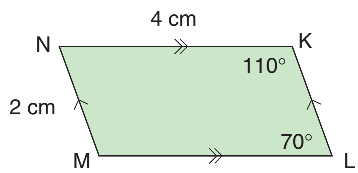
Practice

1. Look at the triangles below.
Find pairs of congruent triangles.
Explain why they are congruent.



2. Are quadrilaterals ABCD and KLMN congruent?
How do you know?

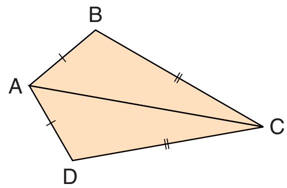


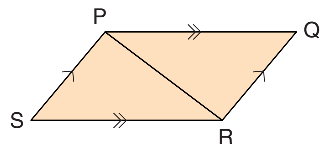


When you name congruent triangles, remember to list corresponding vertices in the same order.

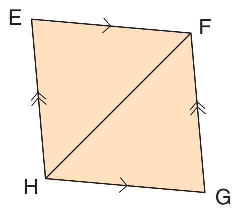
3. In each figure below, name pairs of congruent triangles.
Explain how you know they are congruent.
Try to find more than one way to show
the triangles are congruent.

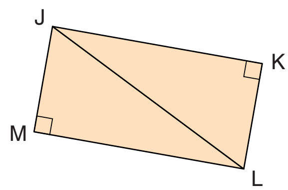
a) ABCD is a kite. **b)** PQRS is a parallelogram.





c) EFGH is a rhombus. **d)** JKLM is a rectangle.





4. For each figure below:
a) Sketch the figure.
b) What are the fewest measurements you need to know to draw the figure?
c) How does your answer to part b help you identify congruent figures of this type?
i) parallelogram **ii)** rectangle **iii)** square

5. $\triangle ABC$ and $\triangle DEF$ have $AB = DE = 6$ cm and $BC = EF = 7$ cm.
a) Sketch the triangles.
b) What else do you need to know to tell if the triangles are congruent?

Mental Math

Which three factors of 24 have a sum of 20?

6. **Assessment Focus** Use dot paper.
   a) Draw two quadrilaterals with equal sides, but the quadrilaterals are not congruent.
   Explain why the quadrilaterals are not congruent.
   b) Use the 4 side lengths in part a. Draw two congruent quadrilaterals with these side lengths.
   Explain how you know the quadrilaterals are congruent.
   c) Explain how the quadrilaterals in parts a and b are different.

7. Alex called a carpet store. He wanted a piece of carpet to repair a damaged rug. Alex asked for a piece measuring 3 m by 4 m by 5 m by 6 m. Explain why the salesperson could not help Alex.

8. a) Are all isosceles triangles with two 50° angles congruent? Explain.
   b) Are all isosceles triangles with two 50° angles and exactly one side of length 10 cm congruent? Explain.

Take It Further

9. Construct a right triangle with one side 5 cm and the longest side 8 cm.
   a) Can you draw two different triangles with those measurements?
   b) If your answer to part a is yes, draw the triangles.
   c) If your answer to part a is no, explain how you know that only one triangle can be drawn with these measurements.

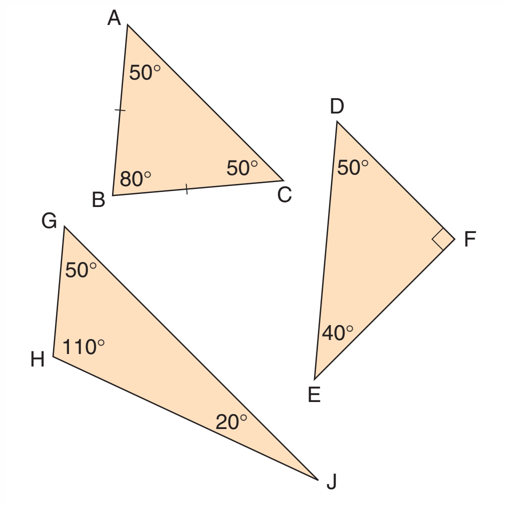
Reflect

Describe the different ways you can tell if two triangles are congruent.

Mid-Unit Review

LESSON

7.1 **1.** Identify each figure.



a) a scalene triangle with an angle of 50°

b) an isosceles triangle with an angle of 50°

c) a right triangle with an angle of 50°

d) an obtuse triangle with an angle of 50°

e) an acute triangle with an angle of 50°

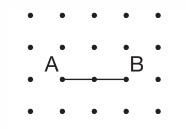
7.1 7.2 **2.** Use dot paper.

a) Draw 2 congruent concave hexagons. How do you know the hexagons are congruent? How do you know they are concave?

b) Draw 2 congruent convex hexagons. How do you know the hexagons are convex? How do you know they are congruent?

7.2 **3.** Segment AB is one side of △ABC. Use dot paper.

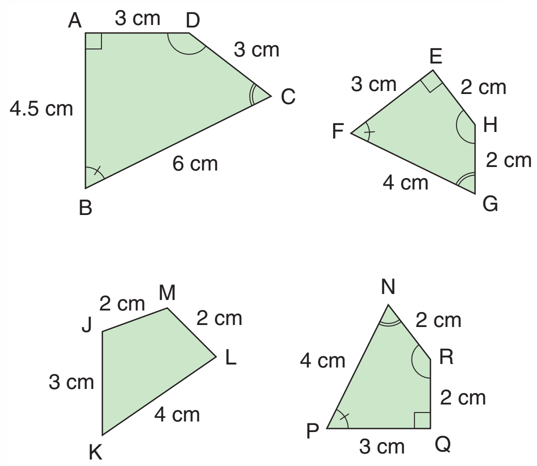
a) Draw △ABC.



b) Draw a triangle congruent to △ABC. How do you know the triangles are congruent?

c) Draw a triangle that is *not* congruent to △ABC. How do you know the triangles are *not* congruent?

4. Use these figures.



a) Identify 2 figures that are *not* congruent. How do you know they are not congruent?

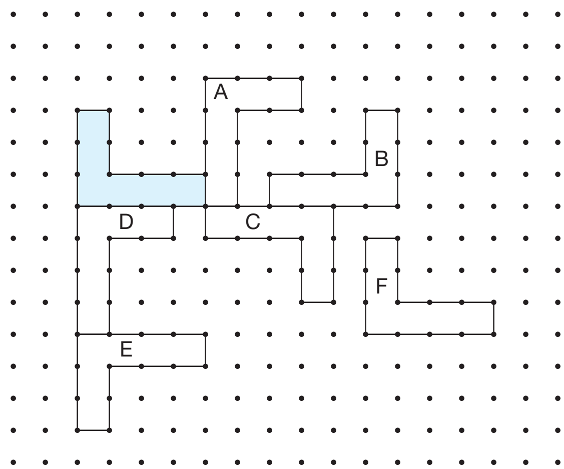
b) Identify 2 congruent figures. How do you know they are congruent?

7.3 Transformations

Focus Recognize transformation images.

Explore

Work with a partner.
Your teacher will give you a large copy of these figures.



Use tracing paper and a Mira if they help.

The shaded figure has been translated, rotated, and reflected.
Each labelled figure is the image after a transformation.
Identify the transformation that produced each image.
Explain how you know.

Reflect & Share

Discuss your strategies for identifying each transformation.
What is special about a reflection image? A translation image?
A rotation image?

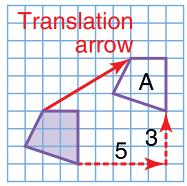
Connect

We can show transformations on a grid.

Translation

The translation image and the shaded figure are congruent.

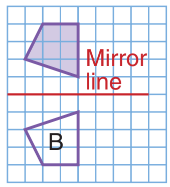
The shaded figure is translated 5 units right and 3 units up.
Its translation image is figure A.
The translation arrow shows the movement in a straight line.



Reflection

The reflection image and the shaded figure are congruent.

The shaded figure is reflected in a horizontal line 1 unit below the figure. Its reflection image is figure B.

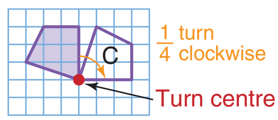


The figures have different orientations. That is, you flip one figure to make it coincide with the other figure.

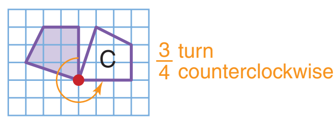
Rotation

The rotation image and the shaded figure are congruent.

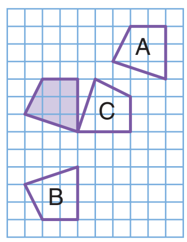
The shaded figure is rotated a $\frac{1}{4}$ turn clockwise. The turn centre is the vertex indicated.
The rotation image is figure C.



We get the same image if the shaded figure is rotated a $\frac{3}{4}$ turn counterclockwise about the turn centre.



Here are the three images and the shaded figure on the same grid.

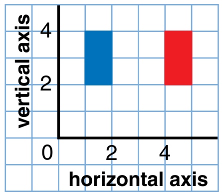


We can show transformations on a coordinate grid.

Example

Look at these rectangles.

Is one rectangle a transformation image of the other? Explain.

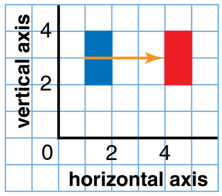


Solution

Let the blue rectangle be the original figure.
And let the red rectangle be the image.

Solution 1

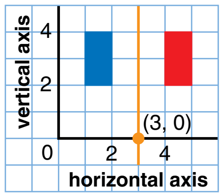
The red rectangle is the image after a translation of 3 units right. The translation arrow shows the movement.



Solution 2

Use a Mira to verify the image.

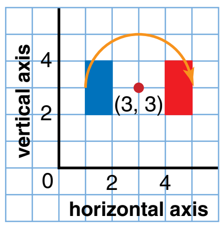
The red rectangle is the image after a reflection in a vertical line through (3, 0) on the horizontal axis.



Solution 3

Use tracing paper to verify the image.

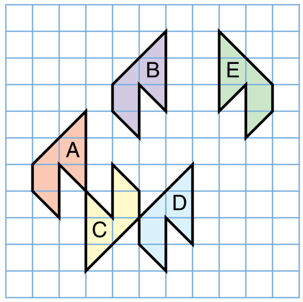
The red rectangle is the image after a rotation of a $\frac{1}{2}$ turn about the point with coordinates (3, 3).



The *Example* shows that an image may be the result of any one of the 3 transformations.
It also shows a rotation about a turn centre that is not on the figure.

Practice

1. Use the figures below.



Identify the transformation for which:

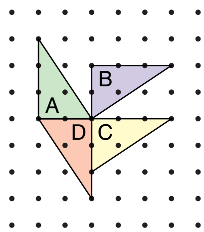
a) Figure B is the image of Figure A.
b) Figure C is the image of Figure A.
c) Figure E is the image of Figure B.
d) Figure A is the image of Figure D.
e) Figure C is the image of Figure D.

Number Strategies

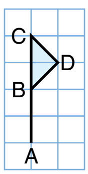
Add or subtract, as indicated.

- $\frac{5}{8} + \frac{7}{6}$
- $\frac{17}{10} - \frac{3}{4}$
- $\frac{11}{12} - \frac{2}{3}$
- $\frac{4}{5} + \frac{5}{6}$

2. Identify each transformation.



a) Figure A is the image of Figure B.
b) Figure B is the image of Figure C.
c) Figure C is the image of Figure D.
d) Figure D is the image of Figure A.

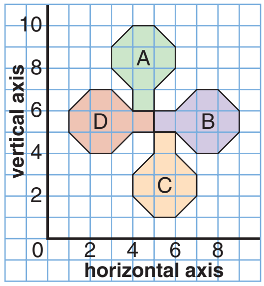


3. Draw this flag on a coordinate grid. The coordinates are A(11, 11), B(11, 13), C(11, 15), and D(12, 14).
Draw the image of the flag after each transformation.

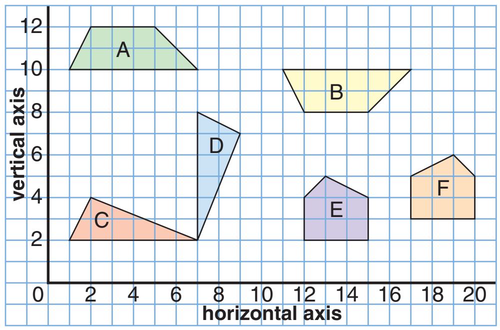
a) a translation 3 units right
b) a translation 5 units down
c) a reflection in a vertical line through (9, 0)
d) a reflection in a horizontal line through (0, 8)
e) a rotation of a $\frac{1}{2}$ turn about point A
f) a rotation of a $\frac{1}{4}$ turn clockwise about point C

Home Quit

4. How many different ways can each figure be described as a transformation of another figure? Explain.



5. **a)** Which pairs of congruent figures do *not* represent a figure and its transformation image? How do you know?

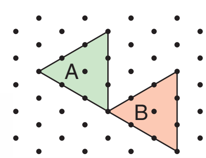


b) For each pair of congruent figures that do show a transformation, identify the transformation.

6. **Assessment Focus** Use grid paper.
In each case, describe the figure you drew.
a) Draw a figure for which a translation image is also a reflection image and a rotation image. Draw the translation image.
b) Draw a figure for which a translation image is also a reflection image, but *not* a rotation image. Draw the translation image.
c) Draw a figure for which a translation image is *not* a reflection image *nor* a rotation image. Draw the translation image.

Take It Further

7. Describe Figure A as a transformation image of Figure B as many different ways as possible.



Reflect

When you see a figure and its transformation image on a grid, how do you identify the transformation?
Use diagrams in your explanation.

7.4 Tiling Patterns

Focus Construct and analyse tiling patterns.

Explore

Work on your own.
You will need index cards, a ruler, and scissors.

> You can rotate or flip the figure to try to make it fit.

➢ Draw a triangle on a card. Cut it out.
Use tracings of the triangle to cover a piece of paper.
➢ Draw a quadrilateral on a card. Cut it out.
Use tracings of the quadrilateral to cover a piece of paper.
➢ Draw a pentagon on a card. Cut it out.
Use tracings of the pentagon to cover a piece of paper.

Reflect & Share

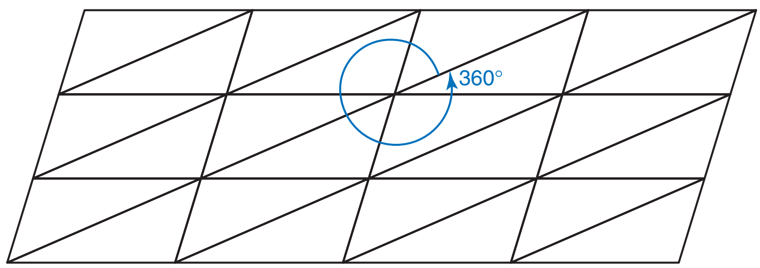
Share your results with the class.
- Will congruent triangles cover a page and leave no gaps? Explain.
- Will congruent quadrilaterals cover a page and leave no gaps? Explain.
- Will congruent pentagons cover a page and leave no gaps? Explain.

How can you tell if congruent figures will cover a page and leave no gaps?

Connect

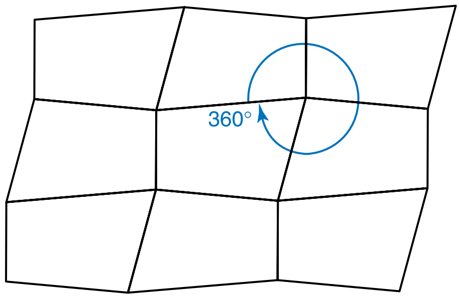
When congruent copies of a figure cover a page and leave no gaps, we say the figure **tiles the plane**.
➢ A triangle always tiles the plane.

> At any point where vertices meet, the angles add to 360°.



➢ A quadrilateral always tiles the plane.

At any point where vertices meet, the angles add to 360°.

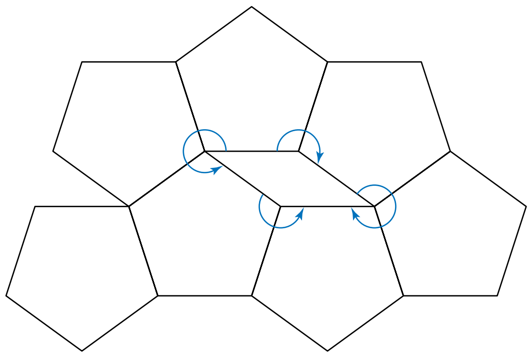


Example

Will a pentagon always tile the plane? Explain.

Solution

If we can find a pentagon that does *not* tile the plane,
we can say that a pentagon does not always tile the plane.
Draw a regular pentagon.
Use tracing paper to repeat the pentagon to try to cover the page.



This pentagon does not cover the page.
It leaves gaps that are rhombuses.
Five vertices do not meet.
There are points where 3 vertices meet and
the sum of the angles is less than 360°.
There are points where 2 vertices meet and
the sum of the angles is less than 360°.
So, a pentagon does not always tile the plane.

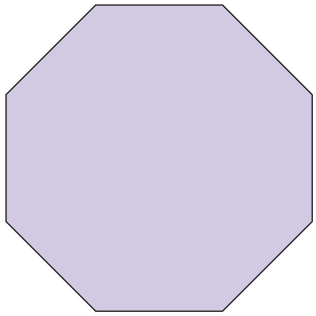
In the *Practice* questions, you will investigate to find which other figures *do* tile the plane.

Practice

1. Use dot paper.
   Draw a convex hexagon that is not regular.
   Try to cover the dot paper with copies of this hexagon.
   Does the hexagon tile the plane? Explain.

2. Use dot paper.
   Draw a concave hexagon.
   Try to cover the dot paper with copies of this hexagon.
   Does the hexagon tile the plane? Explain.

3. Here is a regular octagon.



   Trace this octagon.
   Try to tile the plane.
   What do you notice?

4. Look at the picture called *Reptiles,* drawn by M.C. Escher.

   Which figure do you think Escher started with?
   Explain how Escher's reptiles tile the plane.

Calculator Skills

Evaluate.

- $23.56 + 27.39 \times 4.35$
- $(23.56 + 27.39) \times 4.35$

Why are the answers different?

5. A floor tile is a regular hexagon.
What happens when you try to tile a rectangular floor with a regular hexagon?
Use isometric dot paper to find out.

6. Why do most tiling patterns in floors and patios use squares or rectangles?

7. **Assessment Focus**
Not all pentagons tile the plane. Use grid paper.
- **a)** Find a pentagon that will tile the plane.
  Describe the pentagon.
  Explain how it tiles the plane.
- **b)** How many different pentagons can you find that will tile the plane? Draw each pentagon and show how it tiles the plane.
- **c)** Explain why some pentagons tile the plane, while others do not.

8. In question 3, you discovered that a regular octagon will not tile the plane.
Use grid paper. Find an octagon that will tile the plane.
Explain how it tiles the plane.

Take It Further

9. Think about "tiling" in nature. Which figures are used?

Reflect

How can you tell if a polygon will tile the plane?
Use examples in your explanation.

7.5 Using Transformations to Make Designs

Focus Create and analyse designs using transformations.

Explore

Work on your own.
You will need isometric dot paper.
Choose two or more of these Pattern Blocks.

Make a design to cover a page.
Copy your design on dot paper.
Label each figure in your design.
Explain your design in terms of transformation images.
That is, how do you rotate, translate, or reflect each Pattern Block to generate the design? Write your instructions carefully.

Reflect & Share

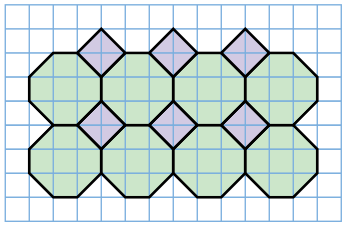
Trade instructions with a classmate.
Generate your classmate's pattern.
Check your version of the pattern with your classmate's.
How do they compare?

Connect

In *Section 7.4*, you investigated tiling patterns.
You used congruent copies of one figure.
You discovered that not all octagons tile the plane.
But an octagon and a square can tile the plane,
as shown in the *Example* that follows.

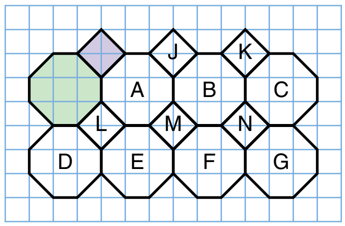
Example

Use transformations to describe how to construct this design.



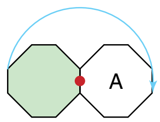
Solution

Label the figures in the design, as shown.



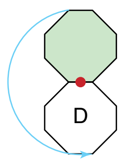
Start with the shaded octagon.

Step 1 To get octagon A, rotate the shaded octagon a $\frac{1}{2}$ turn about a turn centre that is at the midpoint of the right side.



Repeat a similar rotation to get figure B from figure A.

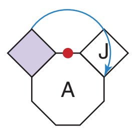
Step 2 To get octagon D, rotate the shaded octagon a $\frac{1}{2}$ turn about a turn centre that is at the midpoint of the bottom side.



Repeat a similar rotation to get octagon E from octagon A.

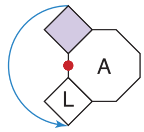
Look at the shaded square.

Step 3 To get square J, rotate the shaded square a $\frac{1}{2}$ turn about the midpoint of the top side of octagon A.



Repeat a similar rotation to get square K from square J.

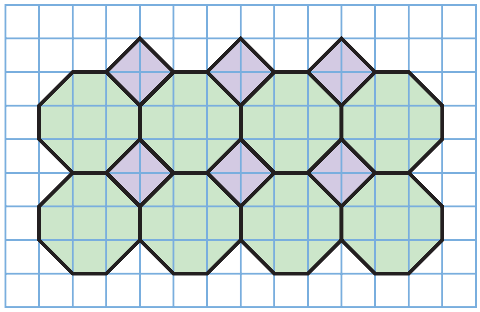
Step 4 To get square L, rotate the shaded square a $\frac{1}{2}$ turn about the midpoint of the left side of octagon A.



Repeat a similar rotation to get square M from square J.

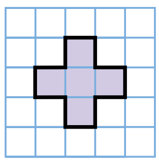
Practice

1. Here is the design from the *Example*.



a) Use translations to describe how to construct this design.
b) Use reflections to describe how to construct this design.

2. Use this figure and transformations to create a design on grid paper.



Describe the design in terms of transformations.

3. Use isometric paper. Use a parallelogram and an equilateral triangle to make a design.
Use transformations to describe the design.

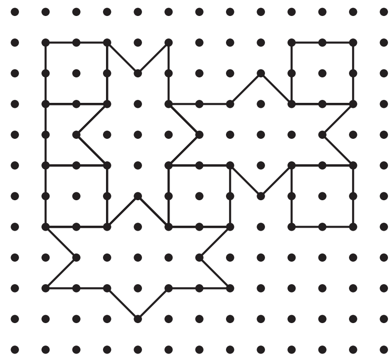
4. Draw a figure. Use transformations of the figure to make a border design for a photo frame.
Draw the design. Describe how you made it.

Number Strategies

Which number is the least?

- the sum of all the factors of 30
- the sum of all the factors of 46
- the product of all the factors of 67

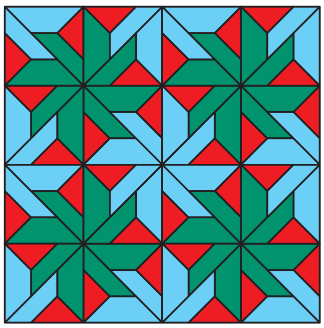
5. The Alhambra is a walled city and fortress in Granada, Spain. It was built in the 14th century. Here is part of one of its many tiling patterns.



Copy this pattern on dot paper.
Continue the pattern to cover the page.
Use transformations to describe the pattern.

6. **Assessment Focus**
Use dot paper or grid paper.
Create a design that uses 2 or more figures that together tile the plane.
Colour your design.
Use transformations to describe your design.
Try to describe your design as many ways as you can.

7. Here is a flooring pattern.



Use a copy of this pattern.
Use transformations to describe the patterns in one square.

Reflect

When you use transformations to describe a design, how do you decide which transformation to use?
Include a design in your explanation.

Using a Computer to Transform Figures

Focus Use technology to create and analyse designs.

Software, such as *The Geometer's Sketchpad*, can be used to transform figures.
Follow these steps:

1. Open *The Geometer's Sketchpad*.
From the File menu, choose **New Sketch**.

To make a "grid paper" screen:

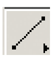
2. From the **Edit** menu, click on **Preferences**.
Select the **Units** tab.
Check that the Distance Units are cm.
Click **OK**.

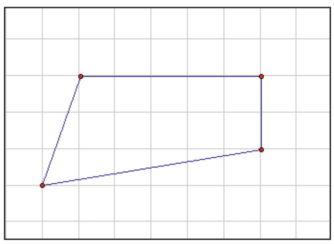
3. From the **Graph** menu, choose **Define Coordinate System**.
The screen has grid lines and two numbered axes.

4. Click on each axis and the two red dots.
The axes and the dots are highlighted.
From the **Display** menu, choose **Hide Objects**.
The axes and dots disappear.
The screen appears like a piece of grid paper.

5. From the **Graph** menu, choose **Snap Points**.

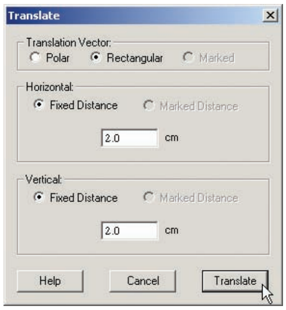
Translating a Figure

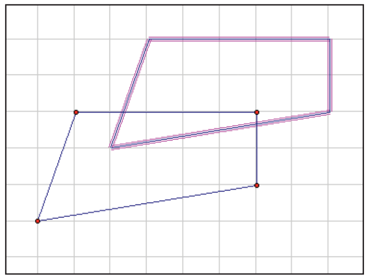
6. To create a quadrilateral:
From the **Toolbox**, choose .
Click and drag to construct a quadrilateral.



7. To translate the quadrilateral:
From the **Toolbox**, choose .
Click each side of the quadrilateral to select it.
The quadrilateral is highlighted.

8. From the **Transform** menu, choose **Translate**.
Under **Translation Vector:**, choose **Rectangular**.
Under **Horizontal:**, choose **Fixed Distance**.
Enter 2.0 cm for the Horizontal distance.
Under **Vertical:**, choose **Fixed Distance**.
Enter 2.0 cm for the Vertical distance (below left).
Click **Translate** to get the quadrilateral and its image after a translation 2 right, 2 up (below right).





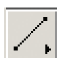
9. Drag any vertex or side of the original figure.
See what happens to the translation image.

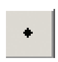
10. From the **Edit** menu, choose **Undo Translate Point**.
The screen shows the original quadrilateral and translation image.
To print the quadrilateral and its translation image, from the **File** menu, choose **Print**.

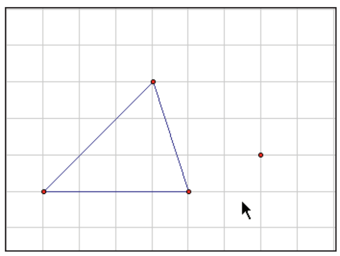
11. Repeat *Steps 7* to *10* using different horizontal and vertical distances.

Rotating a Figure

12. From the **File** menu, choose **New Sketch**.
Follow *Steps 2* to *5* to make a "grid paper" screen.

13. To create a triangle:
From the **Toolbox**, choose .
Click and drag to construct a triangle.

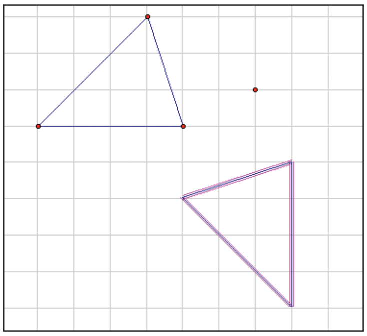
14. To rotate the triangle:
From the **Toolbox**, choose .
Click to place a point near the triangle.



15. Click to select the point.
From the **Transform** menu, choose **Mark Center**.
This is the turn centre for your rotation.

16. From the **Toolbox**, choose .
Click to select each side of the triangle.
The triangle is highlighted.

17. From the **Transform** menu, choose **Rotate**.
Under **Rotate By:**, choose **Fixed Angle**.
Enter 90 degrees. Click **Rotate** to show the triangle and its image after a rotation of 90° counterclockwise.



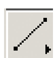
18. Drag any vertex or side of the original figure.
See what happens to the rotation image.

19. From the **Edit** menu, choose **Undo Translate Point**.
The screen shows the original triangle and rotation image.

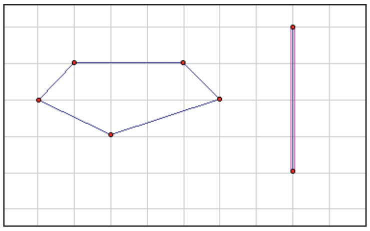
20. Repeat *Steps 16* to *19* using a different number of degrees of rotation. Print the figure and its rotation image.

Reflecting a Figure

21. Repeat *Step 12*.

22. To create a polygon:
From the **Toolbox**, choose .
Click and drag to construct a polygon.

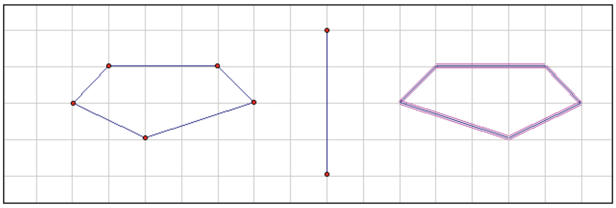
23. To reflect the polygon:
With the Straightedge Tool still selected, draw a vertical line near your polygon. The line is highlighted.



24. From the **Transform** menu, choose **Mark Mirror.**
The line is a mirror line.

25. From the **Toolbox,** choose .
Click to select each side of the polygon.
The polygon is highlighted.

26. From the **Transform** menu, choose **Reflect**.
The polygon and its reflection image are shown.



27. Drag any vertex or side of the original figure.
See what happens to the reflection image.

28. Drag either end point from the mirror line.
See what happens.

29. From the **Edit** menu, choose **Undo Translate Point**.
Do this two times. The screen shows the original polygon and its reflection image.

30. Repeat *Steps 23* to *28* using a horizontal mirror line.
Print the figure and its reflection image.

✓ Check

Use any or all of the transformations above to make a design that covers the screen.
Print your design.

Choosing a Strategy

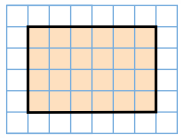
Remember to count squares of different sizes.

Strategies

- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and check.
- Make an organized list.
- Use a pattern.
- Draw a graph.
- Use logical reasoning.

1. A train is travelling at a rate of 1 km every 45 s. At this rate, how far will the train travel in 1 h?

2. How many squares are in this rectangle?



3. Use grid paper. Try to draw a quadrilateral with each number of lines of symmetry.

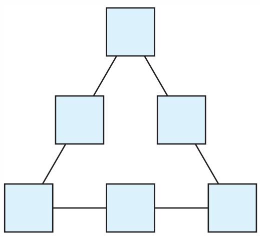
a) 0 **b)** 1 **c)** 2
d) 3 **e)** 4 **f)** 5

Which quadrilaterals could you not draw? Explain.

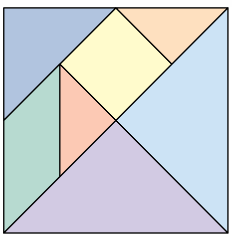
4. Use grid paper. Draw this figure:
It has at least 1 line of symmetry.
It has perimeter 24 units.
It has area 23 square units.

5. Copy the diagram below.
Write these numbers in the boxes: $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1$;
so the sum of the fractions along each side is :

a) $1\frac{1}{2}$ **b)** 2



6. What fraction of the area of a tangram is triangles?



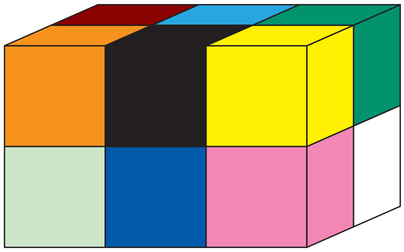
7. Write the next five terms in each pattern. Describe each pattern rule.

a) A (above) B C D (below) E F (above)

b) 1, 5, 10, 25, …

c) O, T, T, F, F, S, S, …

8. This rectangular prism is made with 12 different coloured cubes. The colours are black, white, red, orange, light green, dark green, light blue, dark blue, brown, yellow, pink, and purple.



a) The colours on the left face are light green, orange, brown, and red.
Draw and colour the back face of the prism.

Another congruent prism is made from the coloured cubes. The colours on the front face are: red, orange, dark green, light blue, black, and white.
The colours on the top face are: red, black, white, pink, purple, and yellow.
The colours on the right face side are: light blue, white, yellow, and brown.
The colours on the left face side are: red, dark green, pink, and dark blue.

b) Build the prism. Sketch the prism.

c) What colours are on the bottom face of the prism? How do you know?

Using linking cubes to build the prism.

9. In her fitness program Jessie runs on Mondays, Wednesdays, and Saturdays, and swims on Tuesdays and Fridays. Malcolm runs every third day and swims on the day after each run.
Jessie and Malcolm run together on Saturday July 6.
On what days and dates in July will they:

a) run together again?

b) swim together?

The World of Work

Office Space Planner

An office space planner plans the best use of the office space. He ensures that employees have a workplace that is functional and attractive. Systems furniture is a series of connected partitions, work surfaces, and cabinets. It is frequently used in offices. The planner uses a computer to design multiple 'standard' work areas. Each work area is designed to meet the needs of a group of employees with similar jobs. The work areas are also designed to 'fit' with other work areas of the same size and shape, and with work areas that have different sizes and shapes. It's all a bit of a puzzle! The space planner must solve the puzzle using geometry and an understanding of how people work and interact (ergonomics).

A workplace, where every work area or 'cubicle' is identical, is often seen as 'cell-like' or dehumanizing. Suppose you are an office planner. What might you do to make groups of work areas more appealing, and still make the best use of the available floor area?

Unit Review

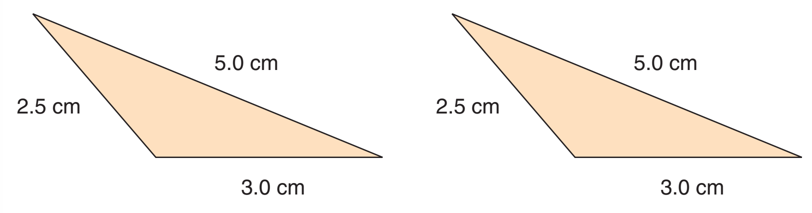
Review any lesson with etext online tutorial

What Do I Need to Know?

✓ Conditions for Congruent Triangles

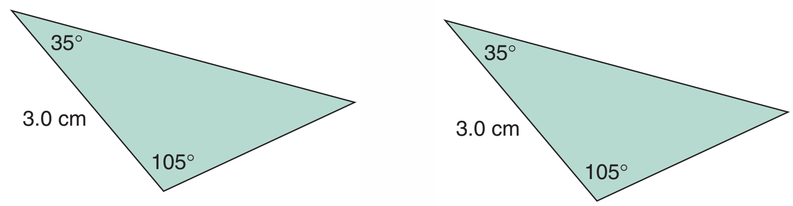
Two triangles are congruent if:

- three pairs of corresponding sides are equal



- two pairs of corresponding sides are equal and the corresponding angles between these sides are equal

- two pairs of corresponding angles are equal and one pair of corresponding sides are equal



✓ Conditions for Congruent Figures

For figures that are not triangles, two figures are congruent if corresponding sides are equal *and* corresponding angles are equal.

What Should I Be Able to Do?

For extra practice, go to page 444.

LESSON

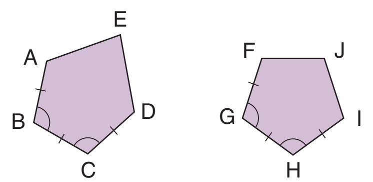
7.1

1. Use grid paper or dot paper.
Draw each figure.
a) a concave hexagon
b) a convex pentagon
c) a concave quadrilateral
d) a figure that is not a polygon
e) a regular triangle
Describe the attributes of each figure. Include angle measures.

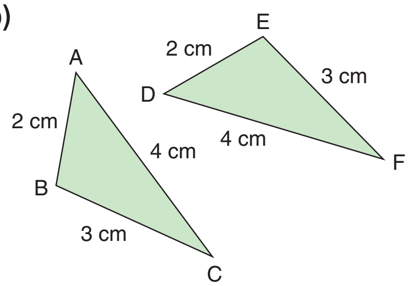
2. Use grid paper or dot paper.
Draw each figure.
a) a hexagon with exactly 2 lines of symmetry
b) a triangle with rotational symmetry of order 3
c) a pentagon with exactly 3 acute angles
d) a pentagon with exactly 3 obtuse angles
Describe the attributes of each figure. Include angle measures.

7.2

3. Are the figures in each pair congruent? How do you know?
a)



b)

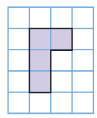


7.3

4. Plot these points on a coordinate grid:
A(4, 6), B(4, 7), C(7, 9), and D(6, 6).
Join the points to form a quadrilateral.
The coordinates of the vertices of 3 images are given.
Identify the transformation that produced each image.
a) C(7, 9), E(5, 12), F(4, 12), G(4, 10)
b) I(4, 16), J(6, 16), K(7, 13), L(4, 15)
c) M(9, 4), N(8, 1), P(6, 1), Q(6, 2)

7.4

5. Copy this figure on grid paper.



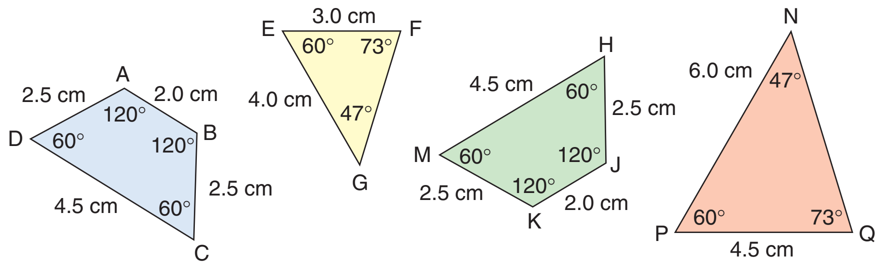
How many different ways can you use the figure to tile the plane?
Show each way you find.

7.5

6. Draw 2 different figures on grid paper that will together tile the plane.
Use the figures to make a design.
Colour one of each figure.
Use transformations to explain how to create the design beginning with each coloured figure.

Practice Test

1. Use the figures below.



Identify:

a) two congruent figures, and explain how you know they are congruent

b) two figures that are *not* congruent, and explain how you know they are not congruent

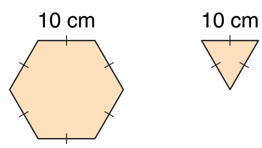
c) a scalene triangle with a 60° angle

d) a quadrilateral with a 60° angle

2. Two triangles are congruent if they have 3 matching sides. Suppose two triangles have 3 matching angles. Are the triangles congruent? Justify your answer.

3. Three students looked at a figure and its transformation image.
Igal said the picture showed a translation.
Shaian said the picture showed a rotation.
Cherie said the picture showed a reflection.
All three students were correct. What might the picture be?
Draw a diagram to show your thinking.

4. Julie will use both of these tiles to cover her floor.



Use isometric paper.
Draw 2 different designs Julie could use.
For each design, use transformations to explain how to create the design.

Unit Problem

Tessellations

When we tile the plane with congruent copies of one figure, we make a **tessellation.**

M.C. Escher was a famous Dutch artist.
He designed many different tessellations.

You will create two designs in the Escher style.
The first design is in the style of *Reptiles*, on page 268.

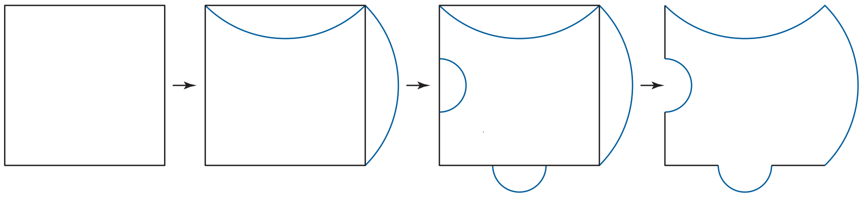
Part 1

Use square dot paper or grid paper.
Tile the plane with a figure of your choice.
Sketch a design on one figure.
Repeat the sketch until every figure in the plane has the design.
Use transformations to describe how to generate the design beginning with one tile.

You could start with a rectangle, parallelogram, or regular hexagon, instead.

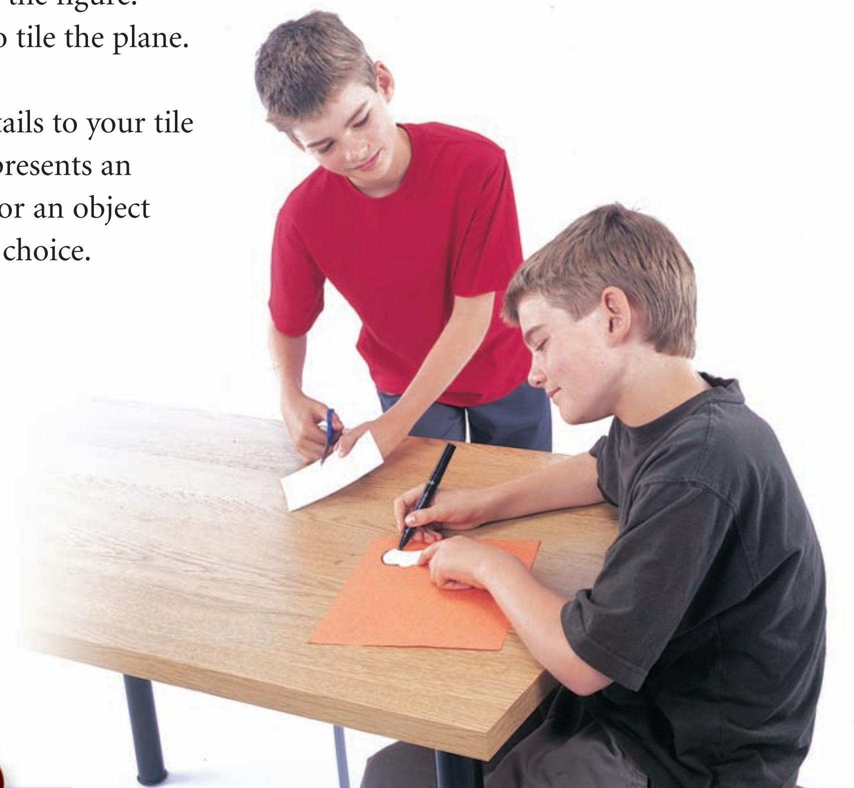
Part 2

Start with a square. Draw congruent curves on 2 sides.
A curve that goes "in" on one side must go "out" on the other side.
Draw different congruent curves on the other 2 sides.



Trace the new figure on cardboard.
Cut out the figure.
Use it to tile the plane.

Add details to your tile so it represents an animal or an object of your choice.



Check List

Your work should show:

- ✓ the initial tile you created for each design
- ✓ the designs you created
- ✓ how you used transformations to create the designs
- ✓ the correct use of mathematical language

Reflect on the Unit

How are transformations related to congruent figures?
Include diagrams in your explanation.