## UNIT

## 4. Fractions and Decimals

Many newspapers and magazines sell advertising space. Why do small companies run small advertisements?

Selling advertising space is a good way to raise funds. Students at Garden Avenue School plan to sell advertising space in their yearbook.

How can fractions and decimals be used in advertising space and advertising rates?

## What

## You'll Learn

- Add and subtract fractions.
- Multiply a fraction by a whole number.
- Compare and order decimals.
- Multiply and divide decimals.
- Use order of operations with decimals.
- Solve problems using fractions and decimals.



## Key Words

- fraction strips
- equivalent fractions
- related
denominators
- unrelated denominators
- common
denominator
- lowest common denominator
- unit fraction
- terminating decimal - repeating decimal


## Skills You'll Need

## Adding and Subtracting Fractions with Pattern Blocks

Let the yellow hexagon represent 1 :

Then the red trapezoid represents $\frac{1}{2}$ :

the blue rhombus represents $\frac{1}{3}$ :

and the green triangle represents $\frac{1}{6}$ :


## Example 1

Use Pattern Blocks to add and subtract.
a) $\frac{1}{3}+\frac{1}{2}$
b) $\frac{1}{2}-\frac{1}{3}$

## Solution

a) $\frac{1}{3}+\frac{1}{2}$
b) $\frac{1}{2}-\frac{1}{3}$


Show $\frac{1}{2}$.
Cover $\frac{1}{3}$.

$\frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

$\frac{1}{6}$ is not covered.

## Check

1. Use Pattern Blocks to add.
a) $\frac{1}{3}+\frac{2}{3}$
b) $\frac{4}{6}+\frac{1}{6}$
c) $\frac{7}{6}+\frac{1}{2}$
d) $\frac{5}{6}+\frac{2}{3}$
2. Use Pattern Blocks to subtract.
a) $\frac{5}{6}-\frac{2}{6}$
b) $\frac{2}{3}-\frac{1}{6}$
c) $\frac{3}{2}-\frac{2}{3}$
d) $\frac{5}{6}-\frac{2}{3}$

## Multiplying by $\mathbf{0 . 1}, \mathbf{0 . 0 1}$, and $\mathbf{0 . 0 0 1}$

We can use patterns to multiply by $0.1,0.01$, and 0.001 .
We can show products on a place-value chart.

|  |  |  |  |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $24 \times 100$ | Thousands | Hundreds | Tens | Ones $\bullet$ Tenths | Hundredths | Thousandths |  |
| $24 \times 10$ |  | 4 | 0 | 0 | $\bullet$ |  |  |
| 2 | 2 | 4 | 0 | $\bullet$ |  |  |  |
| $24 \times 1$ |  |  | 2 | 4 | $\bullet$ |  |  |
| 0.1 |  |  |  | 2 | $\bullet$ | 4 |  |
| $24 \times 0.01$ |  |  |  | 0 | $\bullet$ | 2 | 4 |
| $24 \times 0.001$ |  |  |  | 0 | $\bullet$ | 0 | 2 |

On the chart:

- To multiply by 0.1 , move each digit 1 place to the right.
- To multiply by 0.01 , move each digit 2 places to the right.
- To multiply by 0.001 , move each digit 3 places to the right.

To get the product:

- Move the decimal point 1 place to the left.
- Move the decimal point 2 places to the left.
- Move the decimal point 3 places to the left.


## Example 2

Multiply.
a) $372 \times 0.1$
b) $56 \times 0.01$
c) $41 \times 0.001$

## Solution

Mark the decimal point in the whole number.
a) $372 . \times 0.1=37.2 \quad$ The decimal point moves 1 place to the left.
b) $56 . \times 0.01=0.56$
c) $41 . \times 0.001=0.041$
The decimal point moves 2 places to the left.

Write zeros as placeholders. Then move the decimal point 3 places to the left.

## Check

3. Multiply.
a) $5 \times 0.1$
b) $98 \times 0.1$
c) $124 \times 0.1$
d) $326 \times 0.01$
e) $72 \times 0.01$
f) $6 \times 0.01$
g) $56 \times 0.001$
h) $276 \times 0.001$
i) $8 \times 0.001$

## Operations with Decimals

## Example 3

Evaluate.
a) $82.34+4.7$
b) $79.1-43.8$
c) $426.31 \times 2$
d) $9.47 \div 2$

## Solution

a) $82.34+4.7$

Add the hundredths.
Add the tenths: 10 tenths $=1$ whole
Estimate: $80+5=85$

Add the ones. Add the tens.

$$
8 \frac{1}{2} .34
$$

$$
\begin{gathered}
+4.7 \\
\hline 87.04
\end{gathered}
$$

b) $79.1-43.8$

Estimate: $80-40=40$

To subtract the tenths, trade 1 whole for 10 tenths.
Subtract the ones. Subtract the tens.
$7{ }^{8} 9.11$
$\begin{array}{r}-43.8 \\ \hline 35.3\end{array}$
35.3
c) $426.31 \times 2$

Ignore the decimal point.
Multiply as you would with whole numbers.
Place the decimal point in the answer by estimation:
Estimate: $400 \times 2=800$
426.31
$\times \quad 2$
$\times 852.62$
$426.31 \times 2$ is about $400 \times 2=800$
So, $426.31 \times 2=852.62$
d) $9.47 \div 2$

Use short division.
Divide 9 ones by 2 . There is 1 whole left.
Trade 1 whole for 10 tenths.
Divide 14 tenths by 2.
Estimate: $10 \div 2=5$
$2 \lcm{9 . .^{1} 47^{1} 0}$
4.735

Divide 7 hundredths by 2 . There is 1 hundredth left.
Trade 1 hundredth for 10 thousandths. Divide 10 thousandths by 2 .
So, $9.47 \div 2=4.735$

## Check

4. Evaluate.
a) $12.3+3.5$
b) $21.41-13.8$
c) $31.47 \times 4$
d) $7.44 \div 2$
e) $182.34 \times 7$
f) $52.103+71.81$
g) $49.35 \div 3$
h) $138.97 \times 6$

## Comparing and Ordering Decimals

Here are two ways to order these decimals from least to greatest: $1.84,2.10,1.80,1.89$
$>$ Use a number line.
Mark each decimal on a number line in hundredths from 1.75 to 2.15 .
1.84 is between 1.80 and 1.85 , but closer to 1.85 .
1.89 is between 1.85 and 1.90, but closer to 1.90 .


Use place value.
1.84, 2.10, 1.80, 1.89

In the ones place, the least digit is 1 .
Three decimals have 1 as a ones digit.
Compare these 3 decimals: $1.84,1.80,1.89$
In the tenths place, each decimal has the digit 8.
So, look in the hundredths place:
The least hundredths digit is 0 ; so, 1.80 is the least decimal.
The next hundredths digit is 4 ; so, 1.84 is the next largest decimal.
The next hundredths digit is 9 ; so, 1.89 is the next largest decimal.
So, from least to greatest: $1.80,1.84,1.89,2.10$

## Example 4

Compare each pair of decimals. Place $>$ or $<$ between each pair.
a) 0.5 and 0.08
b) 47.305 and 47.5

## Solution

a) 0.5 and 0.08
Both decimals have 0 ones.
In the tenths place, $5>0$
So, $0.5>0.08$
b) 47.305 and 47.5
Both decimals have 4 tens and 7 ones.
In the tenths place, $3<5$
So, $47.305<47.5$

## Check

5. Order the numbers in each set from least to greatest.
a) $7.32,4.116,3.79,4.12,3.1$
b) $4.4,0.62,2.591,0.65,4.15$
c) $1.25,3.62,1.43,2.81,2.55$
d) $3.669,1.752,3.68,2.67,1.8$

## 4.1 <br> Combining Fractions

Focus Find equivalent fractions and add fractions.

We can use an area model to show fractions of one whole.

## Explore

Work with a partner.
Your teacher will give you a copy of the map.
The map shows a section of land owned by 8 families.

- What fraction of the land did each family own? What strategies did you use?

Four families sold land to the other 4 families.

- Use the clues below to draw the new map.
- Write addition equations, such as $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$, to keep track of the land sales.

1: When all the sales were finished, four families owned all the land - Smith, Perry, Haynes, and Chan.
2: Each owner can walk on her or his land without having to cross someone else's property.
3: Smith now owns $\frac{1}{2}$ of the land.
4: Perry kept $\frac{1}{2}$ of her land, and sold the other half to Chan.
5: Haynes bought land from two other people.
He now owns $\frac{3}{16}$ of the land.
6: Chan now owns the same amount of land as Haynes.


## Reflect \& Share

Did you find any equivalent fractions? How do you know?
Which clues helped you most to draw the new map?
Explain how they helped.

## Connect

This circle shows equivalent fractions.

The circle also shows:
$\frac{1}{2}+\frac{2}{5}+\frac{1}{10}=1$


You can model fractions with strips of paper called fraction strips.
$\square$

Here are more fraction strips and some equivalent fractions they show.


To add $\frac{1}{4}+\frac{1}{2}$, estimate first.
You know that $\frac{1}{2}+\frac{1}{2}=1$. Since $\frac{1}{4}<\frac{1}{2}$, then $\frac{1}{4}+\frac{1}{2}<1$
This strip represents 1 whole.

Use fraction strips to add. Align the strips for $\frac{1}{4}$ and $\frac{1}{2}$.
Find a single strip that has the same length as the two strips.

There are 2 single strips: $\frac{6}{8}$ and $\frac{3}{4}$.
So, $\frac{1}{4}+\frac{1}{2}=\frac{6}{8}$
And, $\frac{1}{4}+\frac{1}{2}=\frac{3}{4}$
$\frac{3}{4}$ and $\frac{6}{8}$ are
equivalent fractions.
They represent the

same amount.

## Example

Use fraction strips to add.
$\frac{1}{3}+\frac{2}{4}$

Solution
$\frac{1}{3}+\frac{2}{4}$

$$
\begin{aligned}
& \text { Estimate: } \frac{1}{3}<\frac{1}{2} ; \frac{2}{4}=\frac{1}{2} \\
& \text { So, } \frac{1}{3}+\frac{2}{4}<1
\end{aligned}
$$

Use the $\frac{1}{3}$ - and $\frac{2}{4}$ -
fraction strips.
These strips align with

the $\frac{5}{6}$-strip.
$\frac{1}{3}+\frac{2}{4}=\frac{5}{6}$

## Practice

## Number Strategies

A loonie has a diameter of about 25 mm . About how many loonies, laid side by side, would there be in 1 km ?

Use models.

1. Which fraction is greater? How do you know?
a) $\frac{2}{3}, \frac{2}{5}$
b) $\frac{3}{4}, \frac{2}{3}$
c) $\frac{5}{8}, \frac{3}{5}$
d) $\frac{3}{4}, \frac{5}{6}$
2. a) Add.
i) $\frac{1}{5}+\frac{1}{5}$
ii) $\frac{2}{3}+\frac{1}{3}$
iii) $\frac{4}{10}+\frac{3}{10}$
iv) $\frac{1}{4}+\frac{2}{4}$
b) Look at your work in part a.

How else could you add fractions with the same denominator?
3. Add. Estimate first.
a) $\frac{1}{5}+\frac{1}{10}$
b) $\frac{1}{2}+\frac{1}{3}$
c) $\frac{1}{6}+\frac{1}{3}$
d) $\frac{1}{4}+\frac{1}{8}$
4. Add. Estimate first.
a) $\frac{2}{4}+\frac{3}{8}$
b) $\frac{2}{3}+\frac{1}{6}$
c) $\frac{2}{5}+\frac{2}{10}$
d) $\frac{3}{6}+\frac{4}{8}$
5. Find 2 fractions that have a sum of 1 .

Try to do this as many ways as you can.
6. Meena's family had a pizza for dinner. The pizza was cut into 8 equal pieces. Meena ate 1 piece, her brother ate 2 pieces, and her mother ate 3 pieces.
a) What fraction of the pizza did Meena eat? Her brother eat? Her mother eat?
b) Which person's fraction can you write in more than one way? Explain.
c) What fraction of the pizza was eaten? What fraction was left?
7. Find the missing number that makes both sides equal.
a) $\frac{1}{5}+\frac{1}{2}=\frac{\square}{10}$
b) $\frac{\square}{10}+\frac{2}{5}=\frac{6}{10}$
c) $\frac{1}{2}+\frac{3}{\square}=\frac{7}{8}$
8. Assessment Focus Boris added 2 fractions. Their sum was $\frac{5}{6}$. Which 2 fractions might Boris have added? Find as many pairs of fractions as you can.

## Reflect

Write 4 equivalent fractions.
How are these fractions the same? How are they different? Explain.

## HOW TO PLAY THE GAME:

1. Use dot paper. Mark an array of 49 dots.

2. Take turns to join 2 dots.
3. The player who completes a square claims that square by writing her initials in it.
4. Continue playing until all squares are claimed.
5. Each player writes the squares he has as a fraction of the whole.
6. The winner is the player who can write the greatest number of equivalent fractions for her share.

## Explore



Work on your own.
Baljit trains for cross-country one hour a day.
One day, she ran for $\frac{1}{3}$ of the time, walked for 25 minutes, then got a second wind and ran for the rest of the time.

How long did Baljit run altogether?
What fraction of the hour is this?

- Use fractions to write an addition equation to show how Baljit spent her hour. Baljit never runs for the whole hour.
- Write another possible training schedule for Baljit. Share it with a classmate.
- Write an additon equation for your classmate's training schedule.


## Reflect \& Share

Compare your equations with your classmate's equation.
Were the equations the same? Explain.
When might a clock be a good model for thinking about adding fractions?
When is a clock not a good model?

## Connect

There are many models that help us add fractions.

- We could use clocks to model halves, thirds, fourths, sixths, and twelfths.


$$
\frac{1}{2}+\frac{1}{3}+\frac{1}{12}=\frac{11}{12}
$$

Circle models are useful when the sum is less than 1.

- When the sum is greater than 1 , we could use fraction strips and a number line.



## Example

Add. $\frac{1}{2}+\frac{4}{5}$
Solution
$\frac{1}{2}+\frac{4}{5}$

Estimate:
$\frac{4}{5}$ is close to 1 ; so, $\frac{1}{2}+\frac{4}{5}>1$

Place both strips end-to-end on the halves line.
The right end of the $\frac{4}{5}$-strip does not line up with a fraction on the halves line.


Place both strips on the fifths line.


The right end of the $\frac{4}{5}$-strip does not line up with a fraction on the fifths line.
Find a line on which to place both strips so the end of the $\frac{4}{5}$-strip lines up with a fraction.


The end of the $\frac{4}{5}$-strip lines up with a fraction on the tenths line.
The strips end at $\frac{13}{10}$.
So, $\frac{1}{2}+\frac{4}{5}=\frac{13}{10}$

## Practice

## Calculator Skills

How many days are there in one million seconds?

Use models.

1. Write 2 equivalent fractions for each fraction shown.
a)

b)

2. Add.
a) $\frac{2}{4}+\frac{3}{4}$
b) $\frac{8}{10}+\frac{9}{10}$
c) $\frac{3}{5}+\frac{4}{5}$
d) $\frac{7}{8}+\frac{7}{8}$
3. Find 2 fractions with a sum of $\frac{3}{2}$.

Try to do this as many ways as you can.
Record each way you find.
4. Add. Estimate first.
a) $\frac{7}{8}+\frac{1}{2}$
b) $\frac{7}{10}+\frac{3}{5}$
C) $\frac{1}{2}+\frac{3}{4}$
d) $\frac{5}{6}+\frac{2}{3}$
5. Add. Estimate first.
a) $\frac{3}{8}+\frac{3}{4}$
b) $\frac{4}{4}+\frac{1}{2}$
c) $\frac{2}{2}+\frac{4}{6}$
d) $\frac{1}{2}+\frac{9}{10}$
6. Use your answers to questions 4 and 5 .
a) Look at the denominators in each part, and the number line you used to get the answer. What patterns do you see?
b) The denominators in each part of questions 4 and 5 are related denominators.
Why do you think they have this name?
7. Add.
a) $\frac{1}{2}+\frac{2}{3}$
b) $\frac{1}{2}+\frac{2}{5}$
c) $\frac{1}{3}+\frac{3}{4}$
d) $\frac{2}{2}+\frac{3}{5}$
8. Look at your answers to question 7.
a) Look at the denominators in each part, and the number line you used to get the answer. What patterns do you see?
b) The denominators in each part of question 7 are called unrelated denominators.
Why do you think they have this name?
c) When you add 2 fractions with unrelated denominators, how do you decide which number line to use?

9. One day Ryan ran for 30 min , rested for 20 min , and then ran for another 45 min . Use fractions of one hour. Write an addition equation that represents his training session.
10. A jug holds 2 cups of liquid. A recipe for punch is $\frac{1}{2}$ cup of orange juice, $\frac{1}{4}$ cup of raspberry juice, $\frac{3}{8}$ cup of grapefruit juice, and $\frac{5}{8}$ cup of lemonade. Is the jug big enough for the punch? Explain.
11. Assessment Focus Use any of the digits
$1,2,3,4,5,6$ only once. Copy and complete. Replace each $\square$ with a number.

a) Find as many sums as you can that are between 1 and 2.
b) Find the least sum that is greater than 1 .

Show your work.
12. Abey and Anoki are eating chocolate bars.

The bars are the same size.
Abey has $\frac{3}{4}$ left. Anoki has $\frac{7}{8}$ left.
How much chocolate is left altogether?

## Take It Further

13. A pitcher of juice is half empty.

After $\frac{1}{2}$ cup of juice is added, the pitcher is $\frac{3}{4}$ full.
How much juice does the pitcher hold when it is full?
Show your thinking.
14. Which number line would you need to add each pair of fractions? Explain.
a) $\frac{1}{3}+\frac{1}{8}$
b) $\frac{1}{3}+\frac{1}{5}$
C) $\frac{1}{4}+\frac{1}{5}$
d) $\frac{1}{4}+\frac{1}{6}$

## Reflect

Which strategies do you use to add 2 fractions?
Include 3 different examples in your answer.

## 4.3 Adding Fractions

In Section 4.2, you used models to add fractions. A clock model only works with certain fractions. You may not always have suitable fraction strips.

We need a new strategy we can use to add fractions without using a model.


## Explore

Work with a partner.


A cookie recipe calls for $\frac{3}{8}$ cup of brown sugar and
$\frac{1}{3}$ cup of white sugar.
How much sugar is needed altogether?
How can you find out?
Show your work.

## Reflect \& Share

Describe your strategy.
Will your strategy work with all fractions?
Test it with $\frac{4}{5}+\frac{2}{3}$.
Use models to justify your strategy.

## Connect

We can use equivalent fractions to add $\frac{1}{4}+\frac{1}{3}$.
Use equivalent fractions that have the same denominators.
12 is a multiple of 3 and 4 .
12 is a common denominator.
$\frac{1}{4}=\frac{3}{12} \quad$ and $\quad \frac{1}{3}=\frac{4}{12}$
So, $\begin{aligned} \frac{1}{4}+\frac{1}{3} & =\frac{3}{12}+\frac{4}{12} \\ & =\frac{7}{12}\end{aligned}$
Both fractions are written


A fraction is in simplest form when the numerator and denominator have no common factors.

Look at the pattern in the equivalent fractions below.



So, to get an equivalent fraction, multiply numerator and denominator by the same number.

We may also get equivalent fractions by dividing.
For example, $\frac{8}{10}$ can be written $\frac{8 \div 2}{10 \div 2}=\frac{4}{5} \quad$ This fraction is in simplest form.

## Example 1

Add. $\frac{5}{6}+\frac{2}{9}$
Solution $\quad \frac{5}{6}+\frac{2}{9}$
The common denominator is a

Estimate:
$\frac{5}{6}$ is close to $1 ; \frac{2}{9}<\frac{1}{2}$
So, $\frac{5}{6}+\frac{2}{9}$ is less than $1 \frac{1}{2}$. multiple of 6 and 9 .
List the multiples of $6: 6,12,18,24, \ldots$
List the multiples of $9: 9,18,27,36, \ldots$
18 is the lowest common multiple of 6 and 9.
So, choose 18 as the common denominator.

| $\frac{5}{6}+\frac{2}{9}=\frac{15}{18}+\frac{4}{18}$ |
| :--- |
| $=\frac{19}{18} \quad$ We can write $\frac{19}{18} \quad$We say that 18 is <br> the lowest common <br> denominator. |

## Example 2

Add. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$
Solution
$\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$
2 is a multiple of 4 .

> Estimate:
> $\frac{2}{3}>\frac{1}{2}$ and $\frac{3}{4}>\frac{1}{2}$
> So, $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$ is greater than $1 \frac{1}{2}$.

So, the common denominator is a multiple of 3 and 4 .
List the multiples of $3: 3,6,9,12,15,18, \ldots$
List the multiples of $4: 4,8,12,16,20,24, \ldots$
12 is the lowest common multiple of 3 and 4 .

So, 12 is the lowest common denominator.


$$
\frac{1}{2}+\frac{2}{3}+\frac{3}{4}=\frac{6}{12}+\frac{8}{12}+\frac{9}{12}
$$

$$
=\frac{23}{12} \quad \text { We can write } \frac{23}{12} \text { as a mixed number: } 1 \frac{11}{12}
$$

## Practice

A fraction with numerator 1 is a unit fraction.

## Mental Math

Multiply.

- $5 \times 27 \times 2$
- $10 \times 3 \times 31$
- $4 \times 9 \times 25$
- $6 \times 20 \times 5$

Which mental math strategies did you use?

1. Add. Use grid paper. Draw a picture to show each sum.
a) $\frac{1}{2}+\frac{1}{3}$
b) $\frac{1}{3}+\frac{1}{5}$
C) $\frac{1}{4}+\frac{1}{5}$
d) $\frac{1}{5}+\frac{1}{6}$
2. Copy and complete. Replace each $\square$ with a number to make a true sentence.
a) $\frac{3}{12}=\frac{\square}{4}$
b) $\frac{3}{4}=\frac{6}{\square}$
c) $\frac{3}{6}=\frac{\square}{4}$
d) $\frac{6}{8}=\frac{15}{\square}$
3. Add. Estimate first.
a) $\frac{4}{5}+\frac{1}{2}$
b) $\frac{3}{4}+\frac{1}{3}$
c) $\frac{2}{3}+\frac{4}{5}$
d) $\frac{2}{3}+\frac{3}{4}$
4. Add.
a) $\frac{2}{3}+\frac{2}{9}$
b) $\frac{1}{6}+\frac{5}{12}$
c) $\frac{3}{8}+\frac{1}{2}$
d) $\frac{3}{4}+\frac{7}{8}$
5. Add.
a) $\frac{5}{6}+\frac{1}{4}$
b) $\frac{1}{6}+\frac{4}{9}$
c) $\frac{7}{10}+\frac{4}{6}$
d) $\frac{3}{4}+\frac{3}{10}$
6. One page of a magazine had 2 advertisements.

One was $\frac{1}{8}$ of the page, the other $\frac{1}{16}$.
What fraction of the page did these occupy?
7. Which sum is greater? How do you know?

$$
\frac{2}{3}+\frac{5}{6} \quad \text { or } \quad \frac{3}{4}+\frac{4}{5}
$$

8. Add.
a) $\frac{3}{8}+\frac{1}{2}+\frac{3}{4}$
b) $\frac{1}{4}+\frac{3}{2}+\frac{2}{5}$
c) $\frac{2}{3}+\frac{5}{6}+\frac{4}{9}$

To add 2 mixed numbers:
Add the whole numbers.
Add the fractions.
Write the sum as a mixed number.

Take It Further
9. Add.
a) $1 \frac{1}{6}+2 \frac{1}{2}$
b) $3 \frac{1}{3}+1 \frac{1}{2}$
c) $4 \frac{1}{6}+2 \frac{3}{8}$
10. A recipe for punch calls for $2 \frac{2}{3}$ cups of fruit concentrate and $6 \frac{3}{4}$ cups of water.
How many cups of punch will the recipe make?

11. Assessment Focus Three people shared a pie.

Which statement is true? Can both statements be true?
Use diagrams to show your thinking.
a) Edna ate $\frac{1}{10}$, Farrah ate $\frac{3}{5}$, and Fran ate $\frac{1}{2}$.
b) Edna ate $\frac{3}{10}$, Farrah ate $\frac{1}{5}$, and Fran ate $\frac{1}{2}$.
12. $\frac{1}{2}+\frac{1}{3}+\frac{1}{6}=1$

Find 3 other fractions with different denominators that add to 1 .
Explain your strategy.
13. Copy this sum. Replace $\square$ with the correct digit.
$2 \frac{1}{4}+1 \frac{\square}{3}=3 \frac{7}{12}$
Explain your strategy.

## Reflect

When you add fractions, and the denominators are different, how do you add?
Give 2 different examples. Use pictures to show your thinking.

## 4.4 Subtracting Fractions Using Models

## Explore



Work with a partner.
You will need fraction strips and number lines.
Find 2 fractions with a difference of $\frac{1}{2}$.
How many different pairs of fractions can you find?
Record each pair.

## Reflect \& Share

Discuss with your partner.
How are your strategies for subtracting fractions the same as your strategies for adding fractions? How are they different?

## Connect

To subtract $\frac{3}{5}-\frac{1}{2}$, think addition: What do we add to $\frac{1}{2}$ to get $\frac{3}{5}$ ?
Try the fifths number line.
Place the $\frac{1}{2}$-strip with its right end at $\frac{3}{5}$.


$$
\begin{aligned}
& \text { Estimate: } \\
& \frac{3}{5}<1 \text {; so, } \frac{3}{5}-\frac{1}{2}<\frac{1}{2}
\end{aligned}
$$

Equivalent fractions:
$\frac{3}{5}=\frac{6}{10}$
$\frac{1}{2}=\frac{5}{10}$

The left end of the strip does not line up with a fraction on the line. Use a number line that has equivalent fractions for halves and fifths. Put the $\frac{1}{2}$-strip on the tenths number line, with its right end at $\frac{6}{10}$.


The left end of the strip is at $\frac{1}{10}$.
So, $\frac{3}{5}-\frac{1}{2}=\frac{1}{10}$

## Example

Use fraction strips and number lines to subtract.
a) $\frac{6}{5}-\frac{4}{5}$
b) $\frac{5}{8}-\frac{1}{4}$

## Solution

a) $\frac{6}{5}-\frac{4}{5}$

Think addition.
What do we add to $\frac{4}{5}$ to get $\frac{6}{5}$ ?
Estimate:
$\frac{6}{5}>1, \frac{4}{5}<1$;

Use the fifths number line because
both denominators are 5 .
Place the $\frac{4}{5}$-strip on the fifths number line with its right end at $\frac{6}{5}$.


The left end of the strip is at $\frac{2}{5}$.
So, $\frac{6}{5}-\frac{4}{5}=\frac{2}{5}$
b) $\frac{5}{8}-\frac{1}{4}$

Think addition.
What do we add to $\frac{1}{4}$ to get $\frac{5}{8}$ ?

Estimate:
$\frac{5}{8}$ is between $\frac{1}{2}$ and 1 , $\frac{1}{4}<\frac{1}{2}$; so , $\frac{5}{8}-\frac{1}{4}<\frac{1}{2}$

Use a number line that shows equivalent fractions for eighths and fourths.
That is, use the eighths number line.
Place the $\frac{1}{4}$-strip on the eighths number line with its right end at $\frac{5}{8}$.


The left end of the strip is at $\frac{3}{8}$.
So, $\frac{5}{8}-\frac{1}{4}=\frac{3}{8}$

## Use models.

1. Subtract.
a) $\frac{3}{4}-\frac{2}{4}$
b) $\frac{4}{5}-\frac{1}{5}$
c) $\frac{2}{3}-\frac{1}{3}$
d) $\frac{5}{8}-\frac{3}{8}$
2. a) Write a rule you could use to subtract fractions with like denominators without using number lines or fraction strips.
b) Write 3 subtraction questions with like denominators.

Use your rule to subtract the fractions.
Use fraction strips and number lines to check your answers.
3. Write the subtraction equation that each number line and fraction strip represent.
a)

b)


## Number Strategies

Find:

- 8 tenths less
than 22.23
- 9 hundredths more than 94.43
- 36 hundredths more than 48.425
- 36 thousandths more than 48.425

4. Subtract. Estimate first.
a) $\frac{3}{8}-\frac{1}{4}$
b) $\frac{5}{6}-\frac{2}{3}$
C) $\frac{5}{4}-\frac{1}{2}$
d) $\frac{7}{10}-\frac{3}{5}$
5. Subtract. Estimate first.
a) $\frac{3}{4}-\frac{1}{2}$
b) $\frac{3}{4}-\frac{3}{8}$
C) $\frac{2}{3}-\frac{1}{6}$
d) $\frac{5}{6}-\frac{1}{2}$
6. Aaron has $\frac{2}{3}$ cup of raisins. He gives Raj $\frac{1}{2}$ cup. How much does Aaron have left?
7. Subtract.
a) $\frac{2}{3}-\frac{1}{4}$
b) $\frac{5}{3}-\frac{1}{2}$
c) $\frac{1}{2}-\frac{1}{5}$
d) $\frac{3}{2}-\frac{1}{3}$
8. Subtract.
a) $\frac{11}{8}-\frac{3}{4}$
b) $\frac{3}{2}-\frac{2}{3}$
c) $\frac{9}{5}-\frac{3}{2}$
d) $\frac{5}{3}-\frac{5}{6}$
9. Subtract.
a) $2-\frac{1}{2}$
b) $1-\frac{3}{5}$
C) $2-\frac{5}{4}$
d) $1-\frac{2}{3}$
10. A cookie recipe calls for $\frac{3}{4}$ cup of chocolate chips. Spencer has $\frac{2}{3}$ cup. Does he have enough? If your answer is yes, explain.
If your answer is no, how much more does Spencer need?
11. Copy and replace each $\square$ with a number, to make each statement correct.
Try to do this more than one way.
a) $\frac{3}{4}-\frac{\square}{\square}=\frac{1}{4}$
b) $\frac{\square}{\square}-\frac{1}{5}=\frac{3}{5}$
c) $\frac{\square}{6}-\frac{2}{\square}=\frac{1}{6}$

12. Assessment Focus Kelly had $\frac{3}{4}$ of a tank of gas at the beginning of the week.
At the end of the week, Kelly had $\frac{1}{8}$ of a tank left.
a) Did Kelly use more or less than $\frac{1}{2}$ of a tank? Explain.
b) How much more or less than $\frac{1}{2}$ of a tank did Kelly use?
13. a) Which of these differences is greater than $\frac{1}{2}$ ?

How do you know?
i) $\frac{5}{6}-\frac{2}{3}$
ii) $\frac{5}{6}-\frac{1}{2}$
iii) $\frac{5}{6}-\frac{1}{6}$
b) Explain how you found your answers to part a. Which other way can you find the fractions with a difference greater than $\frac{1}{2}$ ? Explain.

## Reflect

Which pairs of fractions can you not subtract using the number lines and fraction strips you have? Give 3 examples.

Addition and subtraction are related operations.
You can use what you know about adding fractions to subtract them.

## Explore



Work with a partner.
Use any of the digits $1,2,3,4,5,6$ only once.
Copy and complete. Replace each $\square$ with a number.


Find the least difference greater than 0 .
Which strategies did you use?

## Reflect \& Share

Compare your least difference with that of another pair of classmates. Are the differences the same?
If your answer is no, what is the least difference?

## Connect

To subtract $\frac{3}{4}-\frac{1}{6}$, estimate first.
$\frac{3}{4}$ is between $\frac{1}{2}$ and 1 , and $\frac{1}{6}<\frac{1}{4}$.
So, $\frac{3}{4}-\frac{1}{6}$ is about $\frac{1}{2}$.
Use equivalent fractions to subtract.
Express $\frac{3}{4}$ and $\frac{1}{6}$ with a common denominator.
Find the lowest common denominator.
List the multiples of $4: 4,8, \mathbf{1 2}, 16,20, \ldots$
List the multiples of $6: 6,12,18,24,30, \ldots$
The lowest common denominator is 12 .


$$
\begin{aligned}
\frac{3}{4}-\frac{1}{6} & =\frac{9}{12}-\frac{2}{12} \\
& =\frac{7}{12}
\end{aligned}
$$

Think: 9 twelfths minus 2 twelfths is 7 twelfths.

## Example

Subtract.
a) $\frac{3}{4}-\frac{5}{8}$
b) $\frac{3}{2}-\frac{2}{5}$

Solution
a) $\frac{3}{4}-\frac{5}{8}$

Since 8 is a multiple of 4 ,

Estimate:
Both $\frac{3}{4}$ and $\frac{5}{8}$ are between $\frac{1}{2}$ and 1 ; so, $\frac{3}{4}-\frac{5}{8}<\frac{1}{2}$
common denominator.


$$
\begin{aligned}
\frac{3}{4}-\frac{5}{8} & =\frac{6}{8}-\frac{5}{8} \\
& =\frac{1}{8}
\end{aligned}
$$

b) $\frac{3}{2}-\frac{2}{5}$

List the multiples of 2 :

## Estimate:

$\frac{2}{5}<\frac{1}{2}$; so, $\frac{3}{2}-\frac{2}{5}>1$
$2,4,6,8,10,12, \ldots$
List the multiples of 5 :
$5,10,15,20, \ldots$
The lowest common denominator is 10 .
Write each fraction with a denominator of 10 .


$\frac{3}{2}-\frac{2}{5}=\frac{15}{10}-\frac{4}{10}$ $=\frac{11}{10} \quad$ We can write $\frac{11}{10}$ as the mixed number $1 \frac{1}{10}$.

## Practice

1. Subtract.
a) $\frac{4}{5}-\frac{2}{5}$
b) $\frac{2}{3}-\frac{1}{3}$
c) $\frac{7}{9}-\frac{4}{9}$
d) $\frac{5}{7}-\frac{3}{7}$
2. Subtract. Estimate first.
a) $\frac{5}{8}-\frac{1}{2}$
b) $\frac{4}{9}-\frac{1}{3}$
c) $\frac{3}{2}-\frac{4}{10}$
d) $\frac{5}{3}-\frac{5}{6}$
3. Subtract.
a) $\frac{5}{6}-\frac{2}{9}$
b) $\frac{5}{6}-\frac{2}{4}$
c) $\frac{5}{8}-\frac{3}{12}$
d) $\frac{7}{10}-\frac{4}{15}$
4. Subtract. Estimate first.
a) $\frac{3}{4}-\frac{2}{3}$
b) $\frac{5}{2}-\frac{3}{4}$
c) $\frac{4}{5}-\frac{2}{3}$
d) $\frac{5}{4}-\frac{4}{5}$
5. Subtract.
a) $\frac{4}{6}-\frac{1}{2}$
b) $\frac{5}{3}-\frac{3}{4}$
c) $\frac{7}{2}-\frac{3}{2}$
d) $\frac{5}{6}-\frac{3}{4}$

To subtract 2 mixed numbers: Subtract the whole numbers. Subtract the fractions. Write the difference as a mixed number.

## Calculator Skills

Suppose the 8 key on your calculator is broken. How would you use your calculator to find each answer?

- $18+27$
- 118-85
- $18 \times 27$
- $225 \div 8$

6. Subtract.
a) $5 \frac{5}{7}-1 \frac{2}{7}$
b) $3 \frac{4}{9}-2 \frac{1}{6}$
c) $4 \frac{3}{10}-2 \frac{1}{5}$
d) $4 \frac{3}{5}-2 \frac{1}{2}$
7. Assessment Focus Terri biked $2 \frac{1}{4} \mathrm{~h}$ on Sunday. Terri increased the time she biked by $\frac{1}{4} \mathrm{~h}$ every day. Sam biked $\frac{1}{2} \mathrm{~h}$ on Sunday.
Sam increased the time he biked by $\frac{1}{2} h$ every day.
a) Who will bike longer the next Saturday? Explain.
b) For how much longer will this person bike?
c) What did you need to know about fractions to answer these questions?

8. A recipe calls for $\frac{3}{4}$ cup of walnuts and $\frac{2}{3}$ cup of pecans. Which type of nut is used more in the recipe? How much more?
9. Write as many different subtraction questions as you can where the answer is $\frac{3}{4}$.
10. The difference of 2 fractions is $\frac{1}{2}$. The lesser fraction is between 0 and $\frac{1}{4}$. What do you know about the other fraction?

## Reflect

When you subtract fractions, and the denominators are different, how do you subtract? Give 2 different examples.
Explain your steps.

## Shade One

## HOW TO PLAY THE GAME:

1. Each player chooses a different colour marker.
2. Place the set of fraction cards face down.
3. Player A turns over a card. This is his target fraction.
4. Player A could shade the target fraction on one
 fraction strip, or share the fraction among several fraction strips. However, the total fraction shaded must equal the target fraction.

For example: If your target fraction is $\frac{3}{4}$, you could shade from 0 to $\frac{3}{4}$ on the fourths strip, or 0 to $\frac{6}{8}$ on the eighths strip. Or you could shade several fractions that add up to $\frac{3}{4}$. For example, you could shade $\frac{3}{12}, \frac{2}{8}$, and $\frac{1}{4}$; or you could shade $\frac{2}{5}, \frac{1}{10}$, and $\frac{1}{4}$.
5. Player B turns over the next fraction card.

He repeats Step 4 for his fraction.
6. Play continues with players taking turns to shade the target fraction. If there is not enough of a fraction strip left, and it is impossible to shade the target fraction, that player forfeits his turn. When a player is not able to shade any more fractions, the game is over.
7. The player who shades to complete 1 whole gets 1 point.
8. The person with the most points wins.

## Mid-Unit Review

## LESSON

4.1

1. Add. Use fraction strips to help you.
a) $\frac{2}{3}+\frac{1}{6}$
b) $\frac{2}{4}+\frac{1}{3}$
c) $\frac{2}{8}+\frac{2}{4}$
d) $\frac{2}{10}+\frac{3}{5}$
4.2 2. We know that $\frac{1}{2}+\frac{4}{5}=\frac{13}{10}$. Use this result to find each sum.
a) $1 \frac{1}{2}+\frac{4}{5}$
b) $5 \frac{1}{2}+2 \frac{4}{5}$
2. Use models to add.
a) $\frac{4}{5}+\frac{3}{5}$
b) $\frac{2}{3}+\frac{1}{2}$
c) $\frac{1}{2}+\frac{5}{6}$
d) $\frac{3}{2}+\frac{1}{3}$
4.3
3. Add. Estimate first.
a) $\frac{3}{4}+\frac{5}{6}$
b) $\frac{3}{2}+\frac{2}{3}$
c) $\frac{7}{10}+\frac{2}{5}$
d) $\frac{5}{9}+\frac{5}{6}$
4. Add.
a) $\frac{2}{5}+\frac{3}{2}+\frac{3}{10}$
b) $\frac{3}{8}+\frac{3}{4}+\frac{1}{2}$
c) $\frac{1}{3}+\frac{5}{6}+\frac{2}{9}$
5. Add.
a) $2 \frac{1}{3}+3 \frac{1}{3}$
b) $2 \frac{1}{4}+1 \frac{1}{3}$
c) $1 \frac{3}{4}+3 \frac{3}{8}$
6. Use models to subtract.
a) $\frac{7}{8}-\frac{3}{4}$
b) $\frac{3}{2}-\frac{3}{5}$
c) $\frac{4}{3}-\frac{5}{6}$
d) $\frac{11}{6}-\frac{2}{3}$
7. Peter put $\frac{4}{5}$ cup of chocolate chips into his cookie batter. Samantha put $\frac{7}{8}$ cup into hers. Whose batter contains more chips? How do you know?
8. Alex delivered flyers to $\frac{1}{4}$ of her route on Saturday morning. She delivered to $\frac{2}{3}$ of the route on Saturday afternoon, and the remainder on Sunday.
a) What fraction of her route did Alex deliver on Saturday?
b) What fraction did she deliver on Sunday?

9. Subtract. Estimate first.
a) $\frac{3}{2}-\frac{1}{5}$
b) $\frac{7}{6}-\frac{2}{3}$
c) $\frac{11}{6}-\frac{3}{4}$
d) $\frac{13}{10}-\frac{3}{5}$
10. We know that $\frac{9}{8}-\frac{1}{4}=\frac{7}{8}$.

Use this result to find each difference.
Explain how you did this.
a) $\frac{12}{8}-\frac{1}{4}$
b) $\frac{5}{8}-\frac{1}{4}$
12. Subtract.
a) $3 \frac{5}{8}-2 \frac{3}{8}$
b) $2 \frac{1}{2}-1 \frac{1}{4}$

How many ways can you find this sum?
$2+2+2+2+2+2+2+2+2$
You can use the same strategies in Explore.

## Explore

Work with a partner.
Jan takes $\frac{3}{4} \mathrm{~h}$ to walk to her music lesson.
Jan has a music lesson once a week, for 9 weeks.
How much time does Jan spend walking to her music lessons?

## Reflect \& Share

Compare your strategy for solving the problem with that of another pair of classmates.
Did you get the same answers?
If not, who is correct? Explain.

## Connect

$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=\frac{4}{5}$
All the fractions added are $\frac{1}{5}$.
Repeated addition can be
written as multiplication.
$\frac{1}{5}+\frac{1}{5}+\frac{1}{5}+\frac{1}{5}=4 \times \frac{1}{5}=\frac{4}{5}$


We can show this as a picture.
Similarly: $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$
$=5 \times \frac{3}{4}$
$=\frac{15}{4}$
We can show this as a picture.


## Example 1

Use multiplication to find this sum.
$\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}$
Solution

$$
\begin{aligned}
\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8} & =9 \times \frac{3}{8} \\
& =\frac{27}{8}
\end{aligned}
$$

## Think:

9 times 3 eighths is 27 eighths.

## Example 2

Multiply.
a) $\frac{3}{5} \times 5$
b) $7 \times \frac{5}{6}$

Solution
a) $\begin{aligned} \frac{3}{5} \times 5 & =\frac{15}{5} \\ & =3\end{aligned}$
Think: $\frac{15}{5}$ means $15 \div 5$, which is 3 .
b) $7 \times \frac{5}{6}=\frac{35}{6}$
Think: 7 times 5 sixths is 35 sixths.

## Practice

1. Write each repeated addition as a multiplication question.
a) $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$
b) $\frac{2}{7}+\frac{2}{7}+\frac{2}{7}+\frac{2}{7}+\frac{2}{7}$
c) $\frac{3}{10}+\frac{3}{10}+\frac{3}{10}+\frac{3}{10}$
2. Write each multiplication question as repeated addition.

Draw a picture to show each answer.
a) $5 \times \frac{1}{8}$
b) $\frac{2}{5} \times 3$
c) $4 \times \frac{5}{12}$
3. Multiply. Draw a picture to show each answer.
a) $3 \times \frac{4}{7}$
b) $5 \times \frac{1}{12}$
c) $\frac{2}{15} \times 10$
d) $4 \times \frac{9}{4}$
e) $\frac{2}{5} \times 7$
f) $9 \times \frac{1}{2}$

## Number Strategies

Calculate each answer.
$6000 \div 10 \quad 6000 \times 0.1$
$600 \div 10 \quad 600 \times 0.1$
$60 \div 10 \quad 60 \times 0.1$
$6 \div 10 \quad 6 \times 0.1$
What patterns do you see in the questions and answers?
4. Multiply.
a) $3 \times \frac{4}{5}$
b) $5 \times \frac{7}{10}$
c) $\frac{5}{6} \times 6$
d) $\frac{1}{2} \times 5$
e) $12 \times \frac{7}{12}$
f) $\frac{2}{3} \times 9$
5. It takes $\frac{2}{3} \mathrm{~h}$ to pick all the apples on one tree at Springwater Farms. There are 24 trees. How long will it take to pick all the apples? Show your work.
6. a) Draw a picture to show each product. What is each answer?
i) $4 \times \frac{3}{10}$
ii) $3 \times \frac{4}{10}$
b) How are the questions in part a related?

Write 2 more questions like these.
Find each product. What do you notice?
7. A cookie recipe calls for $\frac{3}{4}$ cup of oatmeal.

How much oatmeal is needed to make 3 batches of cookies?
8. Assessment Focus
a) Draw a picture to show $5 \times \frac{1}{2}$.
b) What meaning can you give to $\frac{1}{2} \times 5$ ?

Draw a picture to show your thinking.
Take It Further

## Reflect

9. Jacob takes $\frac{3}{4} \mathrm{~h}$ to fill one shelf at the supermarket. Henry can fill the shelves in half Jacob's time. There are 15 shelves. Henry and Jacob work together. How long will it take to fill the shelves? Justify your answer.


Draw a picture to show why $4 \times \frac{2}{5}$ is the same as $\frac{2}{5}+\frac{2}{5}+\frac{2}{5}+\frac{2}{5}$. Explain your picture.
Draw a different picture to show the same answer.

## Fractions to Decimals

(\#) Recall that $\frac{1}{10}$ means $1 \div 10$, which is 0.1 as a decimal.
Similarly, $\frac{1}{2}$ means $1 \div 2$, which is 0.5 as a decimal.
0.1 and 0.5 are terminating decimals, because each has a definite number of decimal places.

On a calculator, press: $1 \div 11=$ to display 0.090909091
The calculator rounds up the last digit.
The fraction $\frac{1}{11}$ is a repeating decimal.
The decimal for $\frac{1}{11}$ is $0 . \overline{09}$, with a bar above the 0 and 9 to show they repeat.

A period above an equal sign shows the answer is approximate.

We can round $0 . \overline{09}$ to an approximate decimal.

- $0 . \overline{09} \doteq 0.1$ to 1 decimal place
- $0 . \overline{09} \doteq 0.09$ to 2 decimal places
- $0 . \overline{09} \doteq 0.091$ to 3 decimal places

1. Investigate other unit fractions; that is, fractions with numerator 1.
a) Which unit fractions from $\frac{1}{3}$ to $\frac{1}{20}$ produce terminating decimals? Which produce repeating decimals? Explain why.
b) Investigate other fractions.

For example, how are the decimals for $\frac{1}{6}, \frac{2}{6}, \ldots, \frac{5}{6}$ related?
2. Investigate fractions with greater denominators.

For example, how are the decimals for $\frac{1}{9}, \frac{1}{99}, \frac{1}{999}, \ldots$ related?
3. Which is greater in each case? Justify your answer.
a) $0 . \overline{3}$ or 0.3
b) $\frac{1}{9}$ or 0.11

To multiply 2 whole numbers, we can use Base Ten Blocks. This picture shows the product

$$
\begin{aligned}
20 \times 16 & =100+100+60+60 \\
& =320
\end{aligned}
$$

We can also use Base Ten Blocks to multiply a decimal and a whole number.

The flat represents 1 .


The rod represents 0.1.


This picture shows the product
$2 \times 1.6=1+1+0.6+0.6$

$$
=3.2
$$

In Explore, you will use Base Ten Blocks to multiply 2 decimals.


The small cube represents 0.01 .


## Explore

Work with a partner.
A rectangular tabletop measures 2.4 m by 1.8 m .
Use Base Ten Blocks to find the area of the tabletop.
Record your work on grid paper.

## Reflect \& Share

Compare your answer with that of another pair of classmates.
Did you draw the same picture? Explain.
Did you get the same area? Explain.

## Connect

A rectangular park measures 1.7 km by 2.5 km .
Here are 2 ways to find the area of the park.
> Use Base Ten Blocks.


There are 2 flats: $2 \times 1=2$
There are 19 rods: $19 \times 0.1=1.9$
There are 35 small cubes: $35 \times 0.01=0.35$
The total area is $2+1.9+0.35=4.25$
The area of the park is $4.25 \mathrm{~km}^{2}$.

Use the method for multiplying 2 whole numbers.
The area, in square kilometres, is $1.7 \times 2.5$.
Multiply: $17 \times 25$
17
$\times 25$
85

| 340 |
| :--- |
| 425 |

Estimate to place the decimal point in the answer.
$1.7 \times 2.5$ is about $2 \times 3=6$
So, the product is 4.25 .
The area of the park is $4.25 \mathrm{~km}^{2}$.

## Example 1

Multiply. $5.8 \times 9.7$
Solution
$5.8 \times 9.7$ 58
Multiply: $58 \times 97$
$\times 97$
Estimate to place the decimal point.
406
$5.8 \times 9.7$ is about $6 \times 10=60$. $\frac{5220}{5626}$
So, the product is 56.26 .

## Example 2

Multiply.
a) $0.9 \times 6.8$
b) $0.5 \times 0.4$

Solution
a) $0.9 \times 6.8$

Multiply: $9 \times 68$
$\begin{array}{r}\times 9 \\ \hline 612\end{array}$
Estimate to place the decimal point.
$0.9 \times 6.8$ is about $1 \times 7=7$.
So, the product is 6.12 .
b) $0.5 \times 0.4$

Multiply: $5 \times 4=20$
Estimate to place the decimal point.
$0.5 \times 0.4$ is about $1 \times 0.4=0.4$.
So, the product is 0.20 .

## Practice

1. Write a multiplication equation for each picture.

Each small square represents 0.01 .
a)

b)

2. Use Base Ten Blocks to find each product.
a) $2.6 \times 1.5$
b) $1.4 \times 2.8$
c) $2.7 \times 1.6$
3. Use Base Ten Blocks to find each product.

Record your work on grid paper.
a) $2.3 \times 0.4$
b) $0.6 \times 1.9$
c) $0.8 \times 0.7$
4. Multiply: $36 \times 24$

Use this to find each product. Explain your work.
a) $36 \times 2.4$
b) $3.6 \times 24$
c) $3.6 \times 2.4$
5. Multiply.
a) $4.2 \times 3.7$
b) $8.9 \times 0.3$
c) $0.6 \times 0.9$

## Calculator Skills

Which is greater in each pair?
0.3 or $\frac{1}{3} \quad 0.4$ or $\frac{1}{4}$
0.7 or $\frac{2}{3} \quad 0.09$ or $\frac{1}{11}$

Justify your answer.
6. Carla drives 7.6 km to work. She drives the same distance home. How many kilometres does Carla drive in a 5-day workweek?
7. A rectangular plot of land measures 30.5 m by 5.3 m . What is the area of the plot?
8. a) Multiply.
i) $6.3 \times 1.8$
ii) $4.2 \times 0.7$
iii) $0.8 \times 0.5$
b) Look at the questions and products in part a. What patterns do you see in the numbers of decimal places in the question and the product? How could you use this pattern to place the decimal point in a product without estimating?
9. The product of 2 decimals is 0.36 .

What might the decimals be? Find as many answers as possible.
10. Recall that dividing by 10 is the same as multiplying by 0.1 . Multiply to find:
a) $\frac{1}{10}$ of 25.2
b) $\frac{2}{10}$ of 37.3
c) $\frac{6}{10}$ of 58.7
11. Assessment Focus An area rug is rectangular.

Its dimensions are 3.4 m by 2.7 m .
Show different strategies you can use to find the area of the rug. Which strategy is best? Justify your answer.
12. a) Find each product.
i) $4.8 \times 5.3$
ii) $4.8 \times 0.6$
iii) $0.4 \times 0.6$
b) When you multiply 2 decimals, how does the product compare with the numbers you multiplied? Explain your reasoning.

Take It Further
13. Explain why the sum of the number of digits to the right of the decimal point in the factors of a product equals the number of digits to the right of the decimal point in the product.

When you multiply 2 decimals, how do you know where to place the decimal point in the product? Use examples to explain.

Recall that we can use the same Base Ten Blocks to multiply: $14 \times 18=252 \quad$ and $\quad 1.4 \times 1.8=2.52$


The only difference is the values we assign to the flat, the rod, and the small cube.
In a similar way, we can divide 2 decimals by thinking about whole numbers instead.
So, $252 \div 18=14$
and
$2.52 \div 1.8=1.4$

To divide 2 decimals, ignore the decimal points.
Divide as you would whole numbers, then estimate to place the decimal point in the answer.

## Explore



Work with a partner.
Mark bought 19.5 m of fabric to make costumes for a play.
Each costume needs 2.6 m of fabric.
How many costumes can Mark make?
How much material is left over?
Show your work.

## Reflect \& Share

How did you use division of whole numbers to find your answer?

## Connect

Jan bought 18.9 m of framing to make picture frames.
Each picture needs 1.8 m of frame.
How many frames can Jan make?
How much framing material is left over?

When you divide 2 numbers, the answer is the quotient. $\longrightarrow$

Find $189 \div 18$, then estimate.
$1 8 \longdiv { 1 8 9 . 0 }$
180
90
$\frac{90}{0}$
$189 \div 18=10.5$
Estimate to place the decimal point correctly.
$18.9 \div 1.8$ is about $20 \div 2$, which is 10 .
So, $18.9 \div 1.8$ is 10.5 .
Jan can make 10 frames.
10 frames use $10 \times 1.8 \mathrm{~m}=18 \mathrm{~m}$.
So, the framing material left is $18.9 \mathrm{~m}-18 \mathrm{~m}=0.9 \mathrm{~m}$.
Sometimes when we divide 2 decimals, the quotient is not a terminating decimal.

## Example

Divide.
a) $12.5 \div 0.6$
b) $25.2 \div 4.7$

Solution
a) $12.5 \div 0.6$

6 | $125.0^{2} 0^{2} 0^{2} 0$ |
| ---: |
| 20.833 |

Think: Use short division to find $125 \div 6$.

If we continue to divide, the 3 in the quotient repeats.
So, $125 \div 6=20.8 \overline{3}$
To estimate: $12.5 \div 0.6$ is about $13 \div 1=13$.
So, the quotient is $20.8 \overline{3}$.
b) $25.2 \div 4.7$

47 $\begin{array}{r}5.36 \\ 252.00\end{array}$
235
170
$\frac{141}{290}$ 282
8

Think: Use long division to find $252 \div 47$.

There is 1 decimal place in each number in the question. Continue to divide until there are 2 decimal places in the quotient. That is, calculate the quotient to 1 more decimal place than in the question. Then, round the quotient to 1 decimal place.

Round the quotient to 5.4.
To estimate: $25.2 \div 4.7$ is about $25 \div 5=5$.
So, the quotient is approximately 5.4.

## Practice

## Number Strategies

A rectangular prism has dimensions 6 cm by 4 cm by 8 cm . Find the surface area and volume of this rectangular prism.

1. Look at each division equation on the left.

Estimate each quotient on the right.
a) $234 \div 13=18$
$23.4 \div 1.3$
b) $133 \div 7=19$
$13.3 \div 7$
c) $714 \div 34=21$
$71.4 \div 3.4$
d) $450 \div 18=25$
$4.5 \div 1.8$
e) $51 \div 17=3$
$5.1 \div 1.7$
2. Choose the correct quotient for each division question.

Explain how you know.

## Question

## Possible Quotients

a) $5.95 \div 3.5$
17
$315 \quad 31.5$
0.17
b) $195.3 \div 6.2$
$174 \quad 17.4$
3.15
c) $31.32 \div 1.8$
$12 \quad 1.2 \quad 0.12$
1.74
d) $1.44 \div 0.12$
3. Divide.
a) $8.7 \div 0.3$
b) $2.24 \div 0.7$
c) $10.3 \div 0.6$
4. Divide.
a) $10.92 \div 0.6$
b) $30.42 \div 1.3$
c) $18.56 \div 5.8$

## ILath <br> Iningls

## Measurement

The prefixes in units of length less than 1 m show fractions or decimals of 1 m . One decimetre is $\frac{1}{10}$ or 0.1 m .
One centimetre is $\frac{1}{100}$ or 0.01 m .
One millimetre is $\frac{1}{1000}$ or 0.001 m . One micrometre is $\frac{1}{1000000}$ or 0.000001 m .
5. Divide. Round the quotient to the nearest tenth.
a) $172.5 \div 2.6$
b) $21.68 \div 3.4$
c) $92.8 \div 8.2$
6. Divide. Round to the nearest tenth where necessary.
a) $7.3 \div 0.4$
b) $1.98 \div 1.3$
c) $426.8 \div 3.7$
7. The quotient of 2 decimals is 0.12 . What might the decimals be? Write as many different possible decimal pairs as you can.
8. Assessment Focus Alex finds a remnant of landscaping fabric at a garden store.
The fabric is the standard width, with length 9.7 m .
Alex needs twelve $0.85-\mathrm{m}$ pieces for a garden patio.
a) Will Alex have more fabric than she needs? If so, how much more?
b) Will Alex need more fabric? If so, how much more?
9. The area of a rectangular lawn is $120.4 \mathrm{~m}^{2}$. The width is 5.6 m . What is the length?
10. The question $237 \div 7$ does not have an exact quotient. The first five digits of the quotient are 33857. The decimal point has been omitted. Use only this information and estimation. Write an approximate quotient for each question. Justify each answer.
a) $237 \div 0.7$
b) $2.37 \div 7$
c) $23.7 \div 7$
d) $2370 \div 70$
11. Here is a recipe for bread.

Sam wants to make one-half the amount. Find how much of each ingredient Sam needs.
12. Jack has 2.5 L of juice.

Each day, he drinks 0.4 L .
How many days will it take Jack
to drink the juice?
Justify your answer.

## Reflect

Explain how you decide where to place the decimal point when you divide 2 decimals. Use an example in your explanation.

## Advertising Sales Representative

Magazines and newspapers make money by selling advertising space.

The advertising sales representative contacts companies whose products might be of interest to readers. She offers to sell them various sizes of advertisement space at different rates. When talking about ads smaller than a full page, the sales rep uses fractions to describe them. It's much simpler to talk about a $\frac{2}{3}$-page ad instead of a 0.666667 page ad!

The sales rep tries to sell combinations of ads that can fill pages, with no space left over. A sales rep has sold two $\frac{1}{4}$-page ads and one $\frac{1}{6}$-page ad. She wants to know the possible combinations of ad sizes she can sell to fill the page. What might they be?


We use a base ten, or decimal, number system.
Every whole number is also a decimal.
So, we use the same order of operations for decimals as for whole numbers.

## Explore

Work on your own.
Evaluate this expression: $6 \times(15.9+36.4) \div 4$
Explain each step.

## Reflect \& Share

Compare your solution and answer with that of a classmate.
If your answers are different, who has the correct answer?

## Connect

Here is the order of operations.

- Do the operations in brackets first.
- Then divide and multiply, in order, from left to right.
- Then add and subtract, in order, from left to right.


## Example

Evaluate. $12.4-(4.7+1)+2.4 \times 3-4.8 \div 2$
Solution


## Practice

1. Evaluate.
a) $3.4+4 \times 7$
b) $14-2.2 \times 5$
c) $8-3.6 \div 2$
2. Evaluate.
a) $7.4-3+2.3 \times 4$
b) $4.6+5.1-3.2 \div 2$
c) $16.4-(10.8-3.1)$
d) $23 \times 6.2+4$
e) $81.2-(35.8+2.1)$
f) $85.7 \div 0.4 \times 7$

## Number Sense

A number cube has faces labelled 1 to 6 .
The cube is rolled 75 times.

About how many times will the number 3 show?

The mean time is the sum of the times divided by the number of times.
3. Evaluate.
a) $46.78-6.1 \times 2.3$
b) $75.06 \times(3.45-1.2)$
c) $(98.5+7) \div 2.5$
d) $9.0023 \times 5.1-4.32 \times 6$
e) $8.3+46.2 \div 1.4$
f) $70.56-32.8 \div 4.1$
4. Evaluate.
a) $3.2+5.6 \times 7.2 \div 2.4-9.3$
b) $8.5 \times 7-6.3 \div 9+10.6$
c) $1.35+(5 \times 4.9 \div 0.7)-2.7 \times 2.1$
d) $(4.7-3.1) \times 5-7.5 \div 2.5$
5. Evaluate.
a) $164.5 \div 7 \times 10+7.2$
b) $73.8 \times(3.2+6.8)-14.1 \div 0.2$
6. The cross-country team members ran timed circuits. Here are their times: 15.8 min , $12.5 \mathrm{~min}, 18.0 \mathrm{~min}, 14.2 \mathrm{~min}$, $13.9 \mathrm{~min}, 16.0 \mathrm{~min}, 16.2 \mathrm{~min}$, $17.5 \mathrm{~min}, 16.3 \mathrm{~min}, 15.6 \mathrm{~min}$ Find the mean time.


## 7. Assessment Focus

Evaluate. Show all steps.

$$
0.38+16.2 \times(2.1+4)+21 \div 3.5
$$

## Reflect

Explain why the brackets are unnecessary in this expression:
$(4.2 \times 3.8)-(15.25 \div 6.1)$
Find the answer. Show each step.

## Writing a Math Journal Entry

## THINK about what you did in math.

- Which materials did you use?
- Which problem did you solve?

How did you solve it?

- What did you learn?
- What are you confused about?
-What vocabulary did you use?
- What was challenging? What was easy?


## TALK about what you did in math.

- Work with a partner or in a group.
- Share your ideas. Listen to the ideas of others.
- What information did your partner or group share that was new?
- Ask your partner or group members about any issue that was confusing ... what did they say?


## BRAINSTORM all your ideas.

- Make dot jots of the things you discussed and thought about.
Don't leave anything out ... you can edit it later.
- Think about how you can support your written work. Use graphs, tables, numbers, symbols, diagrams, and so on.



## READ your ideas out loud.

- Listen for missing words.
- Be sure your thoughts are in a logical order.
- Add anything that is missing.
- What is unclear? Why? What would help?
- Add graphs, tables, and so on, that will help explain your written work.


## READ your work to your partner or group.

- Listen to your partner's or group members' ideas.
- Discuss: What was done well? What part was not clear? What would help improve the journal entry?


## REVISE and REWRITE.

- Make the changes that are necessary.
- Rewrite on a new piece of paper if necessary.


## Unit Review

## What Do I Need to Know?

## Adding and Subtracting Fractions

There are 4 types of fraction questions. Look at the denominators.

## Same denominators

For example, $\frac{5}{6}$ and $\frac{2}{6}$
$\frac{5}{6}+\frac{2}{6}=\frac{7}{6} \quad \frac{5}{6}-\frac{2}{6}=\frac{3}{6}$

## Related denominators

For example, $\frac{1}{6}$ and $\frac{1}{12}$


12 is a multiple of 6 , so the lowest common denominator is 12 .
$\frac{1}{6}=\frac{2}{12}$
$\frac{1}{6}+\frac{1}{12}=\frac{2}{12}+\frac{1}{12}=\frac{3}{12} \quad \frac{1}{6}-\frac{1}{12}=\frac{2}{12}-\frac{1}{12}=\frac{1}{12}$

## Partially related denominators

For example, $\frac{1}{6}$ and $\frac{1}{9}$
6 and 9 have the common factor, 3 .
So, list multiples to find the lowest common denominator.
Multiples of $6: 6,12,18,24,30, \ldots$
Multiples of 9: $9,18,27,36, \ldots$
The lowest common denominator is 18 .
$\frac{1}{6}=\frac{3}{18} ; \frac{1}{9}=\frac{2}{18} ; \quad \frac{1}{6}+\frac{1}{9}=\frac{3}{18}+\frac{2}{18}=\frac{5}{18} ; \quad \frac{1}{6}-\frac{1}{9}=\frac{3}{18}-\frac{2}{18}=\frac{1}{18}$

## Unrelated denominators

For example, $\frac{1}{5}$ and $\frac{1}{6}$
5 and 6 have no common factors, so the lowest common
denominator is their product: $5 \times 6=30$
$\frac{1}{5}=\frac{6}{30} ; \frac{1}{6}=\frac{5}{30} ; \frac{1}{5}+\frac{1}{6}=\frac{6}{30}+\frac{5}{30}=\frac{11}{30} ; \quad \frac{1}{5}-\frac{1}{6}=\frac{6}{30}-\frac{5}{30}=\frac{1}{30}$

## Multiplying a Fraction by a Whole Number

This is the same as repeatedly adding the same fraction.
$4 \times \frac{3}{5}=\frac{3}{5}+\frac{3}{5}+\frac{3}{5}+\frac{3}{5}=\frac{12}{5}$

## LESSON

4.1 1. Which fraction is greater? How do you know?
a) $\frac{2}{5}, \frac{2}{6}$
b) $\frac{3}{10}, \frac{1}{2}$
c) $\frac{4}{8}, \frac{3}{6}$
d) $\frac{7}{2}, \frac{7}{3}$
2. Add.
a) $\frac{2}{3}+\frac{1}{6}$
b) $\frac{3}{4}+\frac{2}{8}$
c) $\frac{1}{4}+\frac{3}{6}$
d) $\frac{1}{10}+\frac{3}{5}$
3. Find 2 fractions that add to $\frac{5}{8}$. Find as many pairs of fractions as you can.

4. Add.
a) $\frac{2}{3}+\frac{5}{6}$
b) $\frac{1}{2}+\frac{3}{4}$
c) $\frac{1}{2}+\frac{9}{10}$
d) $\frac{8}{8}+\frac{1}{4}$
5. Adam eats $\frac{3}{10}$ of a pizza. Julie eats $\frac{2}{5}$ of the pizza.
a) How much of the pizza is eaten?
b) How much is left?
4.3
6. Add. Draw a picture to show each sum.
a) $\frac{1}{3}+\frac{2}{5}$
b) $\frac{1}{2}+\frac{3}{8}$
c) $\frac{2}{3}+\frac{3}{10}$
d) $\frac{3}{5}+\frac{1}{4}$
7. Add.
a) $\frac{2}{5}+\frac{5}{6}$
b) $\frac{1}{4}+\frac{2}{5}$
c) $\frac{3}{10}+\frac{5}{6}$
d) $\frac{3}{8}+\frac{2}{3}$
8. Add.
a) $6 \frac{1}{3}+2 \frac{1}{3}$
b) $1 \frac{5}{12}+2 \frac{1}{6}$
c) $2 \frac{3}{10}+3 \frac{1}{5}$
d) $5 \frac{1}{4}+2 \frac{2}{5}$
9. Add.
a) $\frac{2}{3}+\frac{3}{4}+\frac{3}{8}$
b) $\frac{5}{2}+\frac{1}{3}+\frac{2}{5}$
c) $\frac{7}{10}+\frac{3}{4}+\frac{5}{8}$
4.4 10. Subtract.
a) $\frac{1}{2}-\frac{1}{3}$
b) $\frac{7}{10}-\frac{2}{5}$
c) $\frac{3}{4}-\frac{1}{8}$
d) $\frac{5}{6}-\frac{1}{3}$
11. Find 2 fractions with a difference of $\frac{1}{4}$. Find as many pairs of fractions as you can.
Remember to use fractions with different denominators.
4.5 12. Ali drank $\frac{3}{4}$ cup of water. Brad drank $\frac{2}{3}$ cup of water.
a) Who drank more water?
b) How much more water did he drink?

13. Subtract.
a) $\frac{9}{10}-\frac{2}{5}$
b) $\frac{7}{3}-\frac{5}{6}$
c) $\frac{8}{5}-\frac{1}{4}$
d) $\frac{9}{4}-\frac{2}{3}$
14. Subtract.
a) $3 \frac{3}{4}-2 \frac{1}{8}$
b) $4 \frac{4}{5}-2 \frac{2}{3}$
c) $9 \frac{1}{2}-3 \frac{1}{3}$
d) $6 \frac{3}{4}-6 \frac{1}{5}$
15. Multiply. Draw a picture to show each answer.
a) $\frac{1}{4} \times 7$
b) $8 \times \frac{3}{8}$
c) $6 \times \frac{7}{10}$
d) $\frac{4}{5} \times 5$
16. Sasha had 16 tomatoes in his vegetable garden. He gave: Samira $\frac{1}{8}$ of the tomatoes; Sielen $\frac{1}{8}$ of the tomatoes; and Amina $\frac{1}{4}$ of the tomatoes.
a) What fraction of the tomatoes did Sasha have left?
b) How many tomatoes did Sasha have left?
17. Orit spends $\frac{1}{4}$ of her day at school, $\frac{1}{12}$ of her day playing soccer, and $\frac{1}{3}$ of her day sleeping.
How many hours are left in Orit's day?
18. A rectangular park has dimensions 2.8 km by 1.9 km . What is the area of the park?
19. Nuri has 10.8 L of water. He pours 1.5 L into each of several plastic bottles.

a) How many bottles can
Nuri fill?
b) How much water is left over?
20. Delia works at the library after school. She earns $\$ 7.50 / \mathrm{h}$. She usually works 15 h a week.
a) What does Delia earn in a week?
Use estimation to check your answer.
b) One week Delia only works $\frac{1}{2}$ the hours she usually works. What are her earnings that week?
4.9 21. Mr. Statler took his class to the local library 4 times last year. One student had a pedometer. She measured the trip as 1.7 km each way. How far did the students walk back and forth to the library during the year?
22. Evaluate.
a) $5.3+5.1 \div 3$
b) $12.6 \times(1.5+2.5)$
c) $68.9-32.7 \times 2$
23. Evaluate.
a) $5.9+3.7 \times 2.8-1.5 \div 0.5$
b) $3.4 \times 1.9 \div 1.7+7.2 \div 1.2$

## Practice Test

1. Add or subtract.
a) $\frac{5}{4}+\frac{3}{8}$
b) $\frac{3}{2}-\frac{3}{5}$
c) $\frac{11}{12}-\frac{2}{3}$
d) $\frac{4}{9}+\frac{7}{6}$
2. a) Find three pairs of fractions that have a sum of $\frac{3}{5}$.
b) Find three pairs of fractions that have a difference of $\frac{1}{5}$.
3. Add or subtract.
a) $6 \frac{3}{8}-2 \frac{1}{5}$
b) $4 \frac{1}{4}+2 \frac{2}{3}$
4. Lana does yard work. The table shows the approximate time for each job.

For one Saturday, Lana has these jobs:

| Job | Time |
| :--- | :--- |
| Mow small lawn | $\frac{1}{2} \mathrm{~h}$ |
| Mow large lawn | $\frac{3}{4} \mathrm{~h}$ |
| Mow lawn/tidy yard | $1 \frac{1}{2} \mathrm{~h}$ |
| Plant annuals | $2 \frac{1}{2} \mathrm{~h}$ |

- mow 3 small lawns
- mow 1 large lawn
- mow lawn/tidy yard in 2 places
- plant annuals in 1 place

Lana needs travel time between jobs, and a break for lunch. Do you think she will be able to do all the jobs? Justify your answer.
5. a) Evaluate.
i) $\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}$
ii) $7 \times \frac{2}{3}$
iii) $\frac{7}{8} \times 8$
b) Convert each answer in part a to a decimal.
c) Order the decimals from least to greatest.
6. A $6-\mathrm{kg}$ bag of fertilizer covers a rectangular area of 2.5 m by 5.0 m .
a) How many bags of fertilizer are needed for a rectangular lawn measuring 7.5 m by 10.2 m ?
b) A $6-\mathrm{kg}$ bag of fertilizer costs $\$ 15.50$.

How much does it cost to fertilize the lawn?
7. Evaluate. Explain each step.
$12.4 \times(2.9+4.6)+23.7 \div 2.4$

## Unit Problem

The students at Garden Avenue School are preparing a special book for the school's 100th anniversary. They finance the book by selling advertising space to sponsors.

The students sold the following space:

| Full page | $\frac{1}{2}$ page | $\frac{1}{3}$ page | $\frac{1}{4}$ page | $\frac{1}{6}$ page | $\frac{1}{8}$ page |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 1 | 3 | 4 | 5 |

All the advertisements are to fit at the back of the book.
Sam asks: "How many pages do we need for the advertisements?"
Mara asks: "Will the advertisements fill the pages?"
Kathleen asks: "Is there more than one way to arrange these advertisements?"
Can you think of other questions students might ask?

1. Find the total advertising space needed.
2. Sketch the pages to find how the advertisements can be placed. Use grid paper if it helps.

3. Compare your group's sketch with those of other groups. When you made your sketch, what decisions did you make about the shape of each advertisement? Did other groups make the same decisions? Explain.
4. What are the fewest pages needed to display the advertisements? Will there be room for any other advertisements? Explain.
5. What else might students need to consider as they prepare the layout for the book?

Here are the fees for the advertisements.

| Size | Full page | $\frac{1}{2}$ page | $\frac{1}{3}$ page | $\frac{1}{4}$ page | $\frac{1}{6}$ page | $\frac{1}{8}$ page |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Cost $(\$)$ | 500 | 360 | 250 | 200 | 150 | 100 |

6. How much money will students get from the advertisers?
7. Students will do the layout on computer.

The cost to copy and bind 500 books is $\$ 4750$. Use the income from question 6 .
a) What do the students have to charge per book so they do not incur a loss? Justify your answer.
b) The students cannot print fewer than 500 books.

What if they can sell only 350 books?
What do the students have to charge per book so they do not incur a loss? Justify your answer.
c) What if the students decide to charge $\$ 5$ per book?

How many books do the students need to sell so they do not incur a loss? Justify your answer.

## Reflect on the Unit

Choose one operation with fractions and a different operation with decimals. Write an example that illustrates how to carry out each operation.

