UNIT 2

Ratio and Rate

The animal kingdom provides much interesting information. We use information to make comparisons. What comparisons can you make from these facts?

A sea otter eats about $\frac{1}{3}$ of its body mass a day.

A Great Dane can eat up to 4 kg of food a day.

A cheetah can reach a top speed of 110 km/h.

A human can run at 18 km/h.

The heart of a blue whale is the size of a small car.

One in 5000 North Atlantic lobsters is born bright blue.

What You'll Learn

- Understand what a ratio is.
- Find equivalent ratios.
- Compare ratios and use them to solve problems.
- Understand what a rate is.
- Find unit rates.
- Compare rates and use them to solve problems.

Why It's Important

You use ratios and rates to compare numbers and quantities; and to compare prices when you shop.

Key Words

- ratio
- part-to-whole ratio
- part-to-part ratio
- terms of a ratio
- equivalent ratios
- simplest form
- rate
- unit rate
- average speed

UNIT 2

Skills You'll Need

Greatest Common Factor

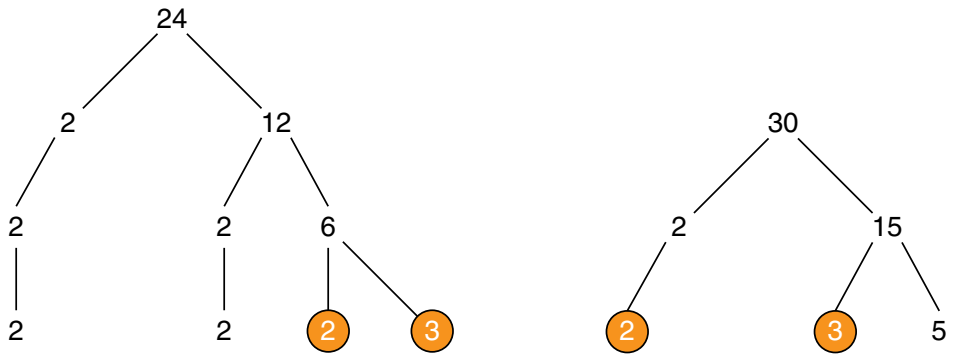
Recall that the greatest common factor (GCF) of a set of numbers is the greatest number that will divide exactly into the given numbers.
For example, 6 is the greatest common factor of 18 and 24.

Example 1

Find the GCF of 24 and 30.

Solution

Draw a factor tree for each number.



Circle the numbers that appear in the bottom row of both factor trees.
2 and 3 are common to both factor trees.
The GCF of 24 and 30 is $2 \times 3 = 6$.

Check

1. Find the GCF of the numbers in each set.

a) 30, 75 **b)** 27, 63 **c)** 42, 56 **d)** 12, 18, 42

Lowest Common Multiple

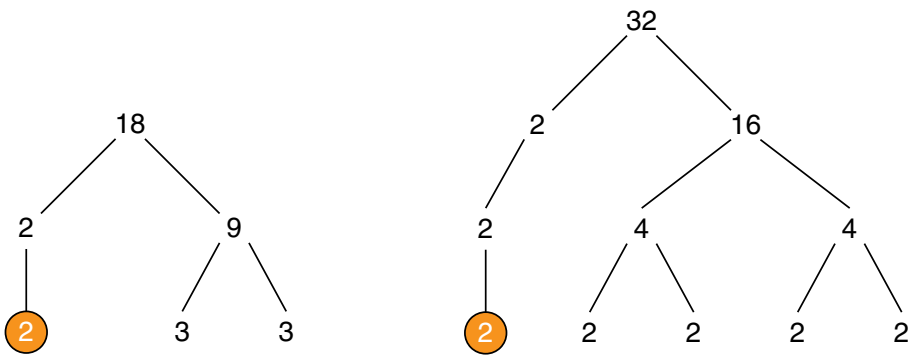
Recall that the lowest common multiple (LCM) of a set of numbers is the least number that is a multiple of each number in the set. This also means that each number in the set is a factor of the lowest common multiple.

Example 2

Find the LCM of 18 and 32.

Solution

Draw a factor tree for each number.



Circle the numbers that appear in the bottom row of both factor trees.
Multiply the numbers not circled and one each of the circled numbers:
$3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 = 288$
The LCM of 18 and 32 is 288.

Check

2. Write the first 6 multiples of each number.
a) 4 **b)** 7 **c)** 9 **d)** 12

3. Find the LCM of the numbers in each pair.
a) 9, 12 **b)** 14, 35 **c)** 16, 40

4. Find the LCM of the numbers in each set.
a) 36, 45 **b)** 3, 4, 6 **c)** 12, 15, 20

Converting among Metric Units

100 cm = 1 m
1000 m = 1 km
1000 g = 1 kg
1000 mL = 1 L

- To convert centimetres to metres, divide by 100.
- To convert:
  metres to kilometres
  grams to kilograms
  millilitres to litres
  } Divide by 1000.
  Divide to convert to a larger unit.
- To convert metres to centimetres, multiply by 100.
- To convert:
  kilometres to metres
  kilograms to grams
  litres to millilitres
  } Multiply by 1000.
  Multiply to convert to a smaller unit.

Example 3

Convert.

a) 650 cm to metres
b) 82 km to metres
c) 2.4 kg to grams
d) 2840 mL to litres

Solution

a) $650 \text{ cm} = \frac{650}{100} \text{ m}$
$= 6.5 \text{ m}$

b) $82 \text{ km} = 82 \times 1000 \text{ m}$
$= 82\,000 \text{ m}$

c) $2.4 \text{ kg} = 2.4 \times 1000 \text{ g}$
$= 2400 \text{ g}$

d) $2840 \text{ mL} = \frac{2840}{1000} \text{ L}$
$= 2.84 \text{ L}$

Check

5. Convert.

a) 1280 cm to metres
b) 680 m to kilometres
c) 2454 g to kilograms
d) 1987 mL to litres
e) 8.2 m to centimetres
f) 1.25 km to metres
g) 0.45 kg to grams
h) 2.3 L to millilitres

2.1 What Is a Ratio?

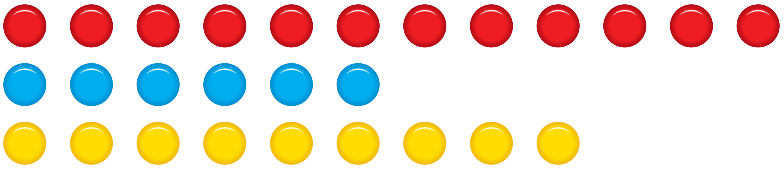
Focus Use models and diagrams to investigate ratios.

There are different ways to compare numbers.
Look at these advertisements.

How are the numbers in each advertisement compared?
Which advertisement is most effective? Explain.

Explore

Work with a partner.



How can you compare the number of blue counters to the number of yellow counters?
How many different ways can you compare the counters?
Write each way you find.

Reflect & Share

Share your list with another pair of classmates.
Add any new comparisons to your list.
Talk about the different ways you compared the counters.

Connect

Here is a collection of models.

This is a **part-to-whole** ratio.

➢ We can use a **ratio** to compare one part of the collection to the whole collection.
There are 9 cars compared to 13 models.
The ratio of cars to models is written as 9 to 13 or 9:13.

This is a **part-to-part** ratio.

➢ We can use a ratio to compare one part of the collection to another part.
There are 9 cars compared to 4 planes.
The ratio of cars to planes is written as 9 to 4 or 9:4.
9 and **4** are called the **terms** of the ratio.
9 is the first term and 4 is the second term.

Example

At a class party, there are 16 boys, 15 girls, and 4 adults.
What is each ratio?

a) boys to girls **b)** girls to adults
c) adults to total number of people at the party

Solution

a) There are 16 boys and 15 girls.
So, the ratio of boys to girls is 16:15.

b) There are 15 girls and 4 adults.
So, the ratio of girls to adults is 15:4.

c) The total number of people is $16 + 15 + 4 = 35$.
So, the ratio of adults to total number of people is 4:35.

Practice

1. Look at the crayons below. Write each ratio.

a) red crayons to the total number of crayons

b) yellow to the total number of crayons

c) blue crayons to green crayons

2. Use words, numbers, or pictures.
Write a ratio to compare the items in each sentence.

a) A student had 9 green counters and 7 red counters on his desk.

b) In a dance team, there were 8 girls and 3 boys.

c) The teacher had 2 fiction and 5 non-fiction books on her desk.

3. The ratio of T-shirts to shorts in Frank's closet is 5:2.
Write the ratio of T-shirts to the total number of garments.

4. **a)** What is the ratio of boys to girls in your class?

b) What is the ratio of girls to boys?

c) What is the ratio of boys to the total number of students in your class?

d) What if two boys leave the room?
What is the ratio in part c now?

5. **a)** Draw two different diagrams to show the ratio 3:5.

b) Draw a diagram to show the ratio 7:1.

6. Maria shares some seashells with Jeff.
Maria says, "Two for you, three for me, two for you, three for me …"
Tonya watches.
At the end, she says, "So Jeff got $\frac{2}{3}$ of the shells."
Do you agree with Tonya? Give reasons for your answer.

Number Strategies

What is the sum of all the prime numbers between 1 and 30?

Take It Further

7. A box contains 8 red, 5 green, 2 brown, 3 purple, 1 blue, and 6 yellow candles.

a) Write each ratio.

i) red:purple **ii)** green:blue

iii) purple:green **iv)** brown and yellow:total candles

b) What if 3 red, 2 green, and 4 yellow candles were burned? Write the new ratios for part a.

8. **Assessment Focus** Patrick plans to make salad. The recipe calls for 3 cups of cooked macaroni, 3 cups of sliced oranges, 2 cups of chopped apple, 1 cup of chopped celery, and 2 cups of mayonnaise.

a) What is the total amount of ingredients?

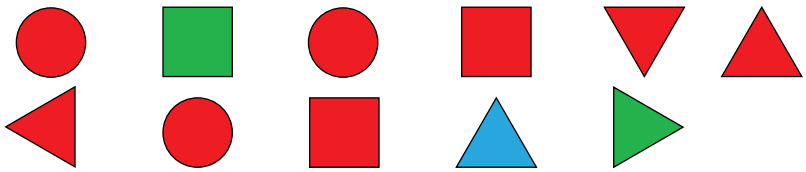
b) What is the ratio of oranges to apples? Mayonnaise to macaroni?

c) What is the ratio of apples and oranges to the total amount of ingredients?

d) Patrick makes a mistake. He uses 2 cups of oranges instead of 3. What are the new ratios in parts b and c?

e) Write your own ratio problem about this salad. Solve your problem.

9. **a)** Create four different ratios using these figures.



b) How can you change one figure to create ratios of 2:5 and 7:3? Explain.

Reflect

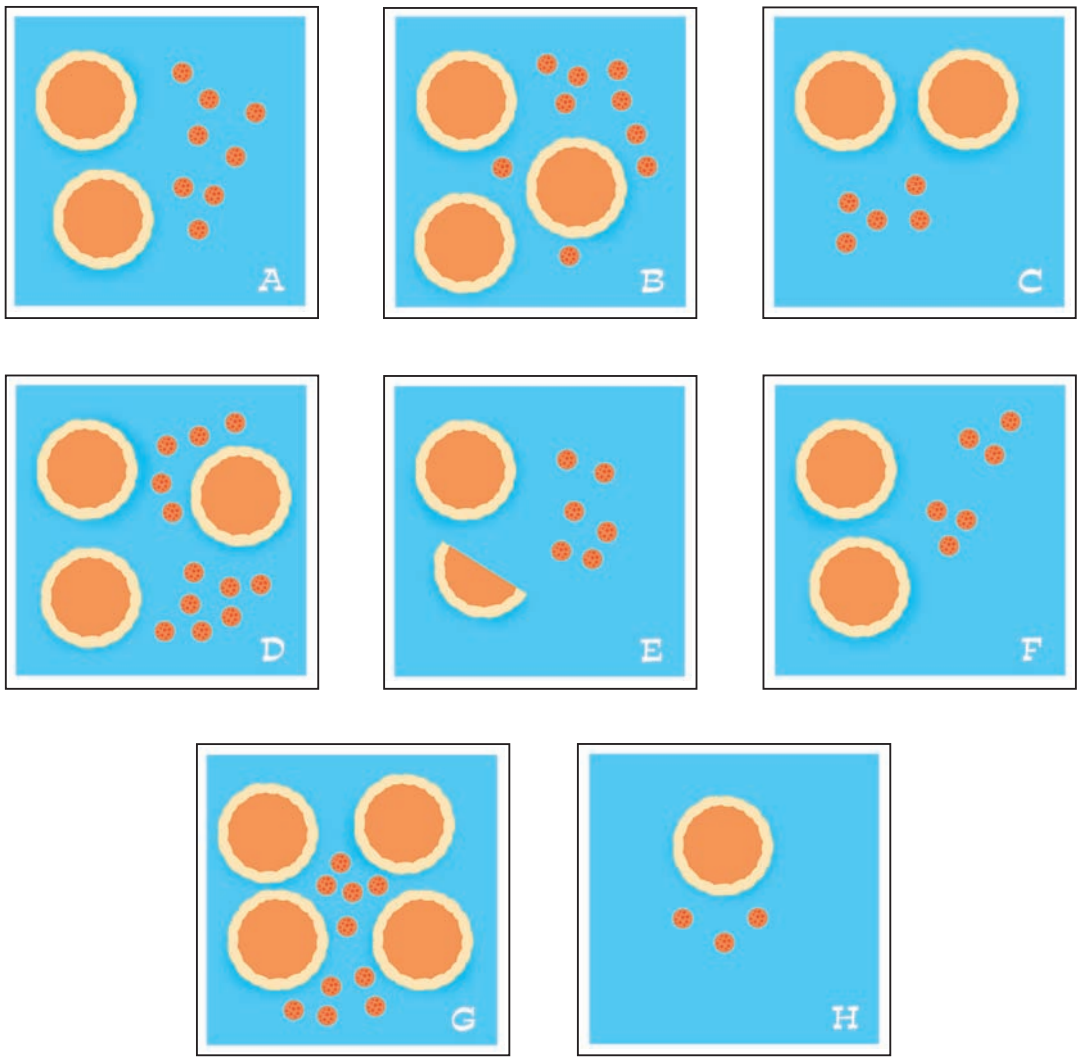
Look in newspapers and magazines for examples of ratios.
Cut out the examples. Paste them in your notebook.
Explain how the ratios are used.
What information can you get from them?

2.2 Equivalent Ratios

Focus Write equivalent ratios.

Explore

Work on your own.
Which cards have the same ratio of pepperoni pieces to pizzas?



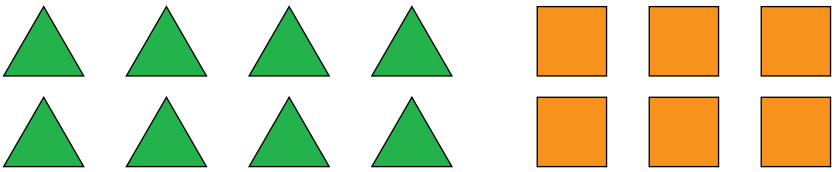
Reflect & Share

Share your answers with a classmate.
Why do you think your answers are correct?
What patterns do you see?

Connect

A ratio of 4:3 means that, for every 4 triangles, there are 3 squares.

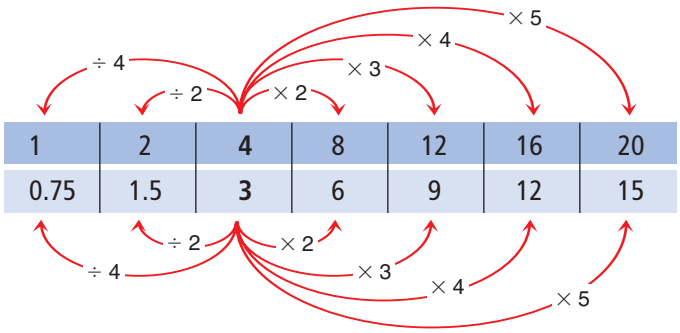
A ratio of 8:6 means that, for every 8 triangles, there are 6 squares.



The ratios 8:6 and 4:3 are called **equivalent ratios**.
Equivalent ratios are equal. 8:6 = 4:3

➢ An equivalent ratio can be formed by multiplying or dividing the terms of a ratio by the same number.

Note that
3 ÷ 2 is $\frac{3}{2}$ = 1.5
and
3 ÷ 4 is $\frac{3}{4}$ = 0.75

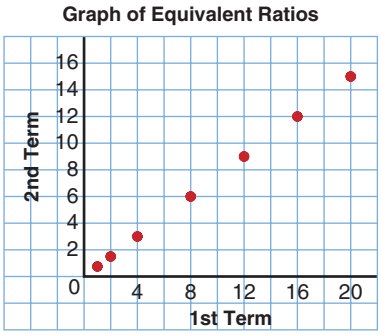


The equivalent ratios are:
1:0.75; 2:1.5; 4:3; 8:6; 12:9; 16:12; 20:15

➢ The equivalent ratios can be shown on a grid.

1st term	1	2	4	8	12	16	20
2nd term	0.75	1.5	3	6	9	12	15

The points representing the ratios lie on a straight line.



A ratio is in simplest form when its terms have no common factors.

➢ When we divide the terms in a ratio by their greatest common factor, we write the ratio in **simplest form**.

To write 24:16 in simplest form:

24:16 The GCF is 8. Divide by 8.

$= (24 \div 8):(16 \div 8)$

$= 3:2$

So, 24:16 and 3:2 are equivalent ratios.

The ratio 3:2 is in simplest form.

Example

Construction kits come in different sizes.

The Regular Kit contains 120 long rods, 80 short rods, and 40 connectors.

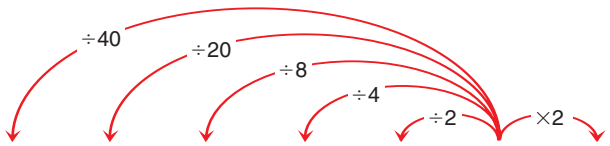
a) What other kits could be created with the same ratio of rods and connectors?

b) One kit has 10 connectors. How many short and long rods does it have?

Solution

a) Use a table to find equivalent ratios.

Label each new kit.



Component	Kit A	Kit B	Kit C	Kit D	Kit E	Regular Kit	Kit F
Long Rods	3	6	15	30	60	120	240
Short Rods	2	4	10	20	40	80	160
Connectors	1	2	5	10	20	40	80

b) Use the table in part a.

The kit with 10 connectors is Kit D.

It has 20 short rods and 30 long rods.

Practice

Number Strategies

Find the greatest common factor of the numbers in each set.

- 16, 40, 24
- 33, 77, 88
- 45, 75, 30
- 150, 75, 225

1. Write three ratios equivalent to each ratio.
Use tables to show your work.

a) 3:4 **b)** 14:4

2. Rewrite each sentence as a ratio statement in simplest form.

a) In a class, there are 15 girls and 12 boys.

b) In a parking lot, there were 4 American cars and 12 Japanese cars.

c) A paint mixture is made up of 6 L of blue paint and 2 L of white paint.

d) A stamp collection contains 12 Canadian stamps and 24 American stamps.

3. Name the pairs of equivalent ratios:
2:3, 9:12, 8:5, 1:2, 2:1, 16:10, 3:6, 6:9, 5:8, 3:4
Tell how you know they are equivalent.

4. In a class library, 3 out of 4 books are non-fiction.
The rest are fiction.

a) How many non-fiction books could there be?
How many fiction books?

b) How many different answers can you find for part a?
Which answers are reasonable? Explain.

5. The official Canadian flag has a length to width ratio of 2:1.
Doreen has a sheet of paper that measures 30 cm by 20 cm.
What are the length and width of the largest Canadian flag Doreen can draw? Sketch a picture of the flag.

6. **Assessment Focus** Use red, blue, and green counters.
Make a set of counters with these two ratios:
red:blue = 5:6 blue:green = 3:4
How many different ways can you do this?
Record each way you find.

Reflect

Choose a ratio. Use pictures, numbers, or words to show how to find two equivalent ratios.

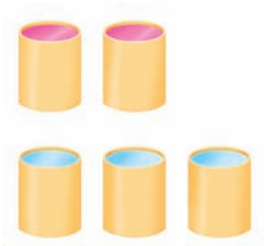
2.3 Comparing Ratios

Focus Use equivalent ratios to compare ratios.

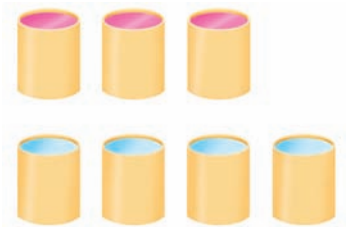
Explore

Work with a partner.

Recipe A for punch calls for 2 cans of concentrate and 3 cans of water.



Recipe B for punch calls for 3 cans of concentrate and 4 cans of water.



In which recipe is the punch stronger?
Or are the drinks the same?
Explain how you know.

Reflect & Share

Compare your answer with that of another pair of classmates.
Compare strategies.
If your answers are the same, which strategy do you prefer? Would there be a situation when the other strategy would be better? Explain.
If your answers are different, find out which is correct.

Connect

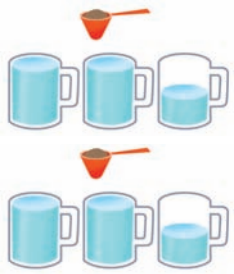
Erica makes her coffee with 2 scoops of coffee to 5 cups of water.

Jim makes his coffee with 3 scoops of coffee to 7 cups of water.

Here are two ways to find out which coffee is stronger.

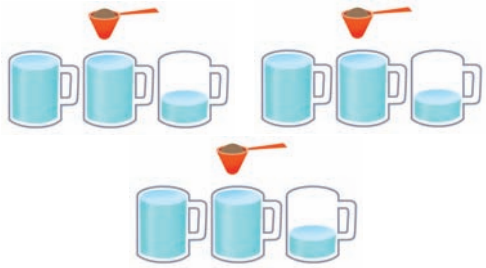
➢ Find how much water is used for 1 scoop of coffee.

Erica



1 scoop of coffee to $2\frac{1}{2}$ cups of water.

Jim

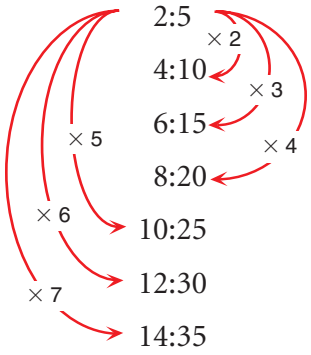


1 scoop of coffee to $2\frac{1}{3}$ cups of water.

Since $2\frac{1}{3}$ is less than $2\frac{1}{2}$,
Jim uses less water to 1 scoop of coffee.
So, Jim's coffee is stronger.

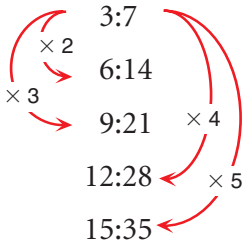
➢ Find how much coffee is used for the same amount of water.
Write each mixture as a ratio.
Write each ratio with the same second term,
then compare the first terms.
Use equivalent ratios.

Erica



Since 2:5 = 14:35,
Erica uses 14 scoops of coffee to 35 cups of water.

Jim



Since 3:7 = 15:35,
Jim uses 15 scoops of coffee to 35 cups of water.

Jim uses more coffee. So, Jim's coffee is stronger.
Notice that, as we multiply to get equivalent ratios,
we get multiples of the terms of the ratios.
That is, the first terms in Erica's equivalent ratios are multiples of 2;
the second terms are multiples of 5.

A quicker way to write each ratio with the same second term is to find the lowest common multiple of the second terms.
That is, the lowest common multiple of 5 and 7 is 35.

Example

At the outdoor centre, there were the same numbers of boys and girls. Five out of every 8 boys wanted to kayak. Two out of every 3 girls wanted to kayak. Do more boys than girls want to kayak? Explain.

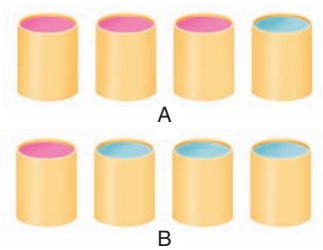
Solution

Five out of every 8 boys want to kayak.
This is a ratio of 5:8.
Two out of every 3 girls want to kayak.
This is a ratio of 2:3.
To compare the ratios, write them with the second terms the same.
The lowest common multiple of 8 and 3 is 24.
Multiply to make the second term of each ratio 24.

Boys	**Girls**
5:8	2:3
$= (5 \times 3):(8 \times 3)$	$= (2 \times 8):(3 \times 8)$
$= 15:24$	$= 16:24$
15 out of 24 boys want to kayak	16 out of 24 girls want to kayak

Since 16 is greater than 15, more girls want to kayak.

Practice



1. The concentrate and water in each picture are mixed.
Which mixture is stronger: A or B?
Draw a picture to show your answer.

2. Two boxes contain pictures of hockey and basketball players.
In one box, the ratio of hockey players to basketball players is 4:3.
In the other box, the ratio is 3:2.
The boxes contain the same number of pictures.

a) What could the total number of pictures be?

b) Which box contains more pictures of hockey players?
Draw a picture to show your answer.

Number Strategies

There are 10 coins
that total $0.60.
What are the coins?

3. In a basketball game, Alison made 6 of 13 free shots.
Madhu made 5 of 9 free shots. Who played better? Explain.

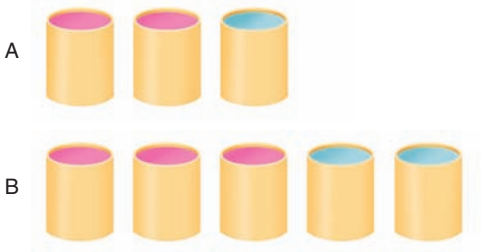
4. The principal is deciding which shade of blue to have the classrooms painted. One shade of blue requires 3 cans of white paint mixed with 4 cans of blue paint. Another shade of blue requires 5 cans of white paint mixed with 7 cans of blue paint.

a) Which mixture will give the darker shade of blue? Explain.

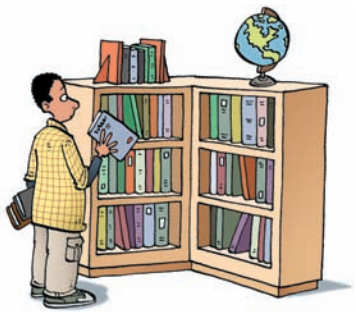
b) Which mixture will require more white paint?

5. Look at the two mixtures.

a) What is the ratio of concentrate to water in A and in B?



b) Explain how you could add concentrate or water to make both ratios the same.
Draw a picture to show your answer.



6. **Assessment Focus** The ratio of fiction to non-fiction books in Ms. Arbuckle's class library is 7:5.
The ratio of fiction to non-fiction books in Mr. Albright's class library is 4:3.
Each classroom has 30 non-fiction books.

a) Which room has more fiction books? How many more?

b) Mr. Albright added two non-fiction books to his class library. Does this make the ratio the same in both classes? Explain.

7. At Ria's party, there were 2 pizzas for every 3 people.
At Amin's party, there were 5 pizzas for every 7 people.
At which party did each person get more pizza? Explain.

Reflect

In one store, the ratio of DVDs to videos is 7:5.
In another store, the ratio of DVDs to videos is 4:3.
Explain why you cannot say which store has more DVDs.

Mid-Unit Review

LESSON

2.1 **1.** In the school parking lot, there are 4 Japanese cars, 7 American cars, 3 German cars, and 2 Korean cars. Write each ratio.

a) American cars to Japanese cars

b) German cars to Korean cars

c) American cars to total number of cars

d) German cars to Japanese cars and American cars

2. A box contains 5 cream chocolates, 2 chocolate-covered almonds, and 3 caramel chocolates.

a) What is each ratio? Sketch a picture for each ratio.

i) almond chocolates to caramel chocolates

ii) cream chocolates to caramel chocolates

iii) cream chocolates to all chocolates

b) Lesley ate one of each kind of chocolate. What is each new ratio for part a?

3. In Mary's closet, there are 7 T-shirts, 4 pairs of shorts, and 3 sweaters. Write each ratio.

a) T-shirts to shorts

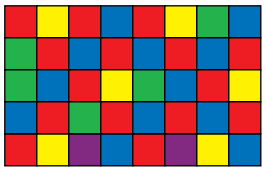
b) sweaters to shorts

c) sweaters to T-shirts and shorts

2.2 **4.** For each ratio, write three equivalent ratios.

a) 5:3 **b)** 6:24

5. **a)** Write each ratio below in simplest form.



i) green squares to red squares

ii) yellow squares to purple squares

iii) red squares to total number of squares

b) State the colours for each ratio.

i) 1:6 **ii)** 2:5

6. In Ms. Bell's class, the ratio of boys to girls is 5:4.

a) There are 15 boys in the class. How many girls are there?

b) Two girls leave. What is the new ratio of boys to girls?

2.3 **7.** A jug of orange juice requires 3 cans of orange concentrate and 5 cans of water.

a) Accidentally, 4 cans of concentrate were mixed with 5 cans of water. Is the mixture stronger or weaker than it should be? Explain.

b) Suppose 6 cans of water were mixed with 3 cans of concentrate. Is the mixture stronger or weaker than it should be? Explain.

2.4 Applications of Ratios

Focus Use ratios to solve problems.

Explore

Work on your own.
In the book *Gulliver's Travels*, Gulliver meets little people who are only 15 cm tall, and giants who are 18 m tall.
Gulliver is 1.80 m tall.
How many times as big as a little person is Gulliver?
How many times as big as Gulliver is a giant?
Write these comparisons as ratios.

Reflect & Share

Compare your answers with those of a classmate.
Work together to find the ratio of the height of the giants to the height of the little people.
How many times as big as a little person is a giant?

Connect

You can use diagrams and tables to model and solve ratio problems.

Example 1

Jolene makes a scale drawing of her home.
She uses a scale of 5 cm to represent 1.5 m.

a) What is the ratio of a length in the drawing to the actual length? What does this ratio mean?

b) Jolene measures her bedroom on the drawing. It is 16 cm long. What is the actual length of Jolene's bedroom?

c) The house measures 18 m by 12 m. What are the dimensions of the scale drawing?

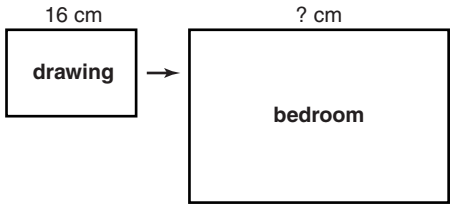
Solution

a) Length in the drawing:actual length
$= 5 \text{ cm}:1.50 \text{ m}$
$= 5 \text{ cm}:150 \text{ cm}$
$= (5 \text{ cm} \div 5 \text{ cm}):(150 \text{ cm} \div 5 \text{ cm})$
$= 1:30$

When you write ratios of measurements, the units must be the same. Multiply 1.50 m by 100 to change metres to centimetres.

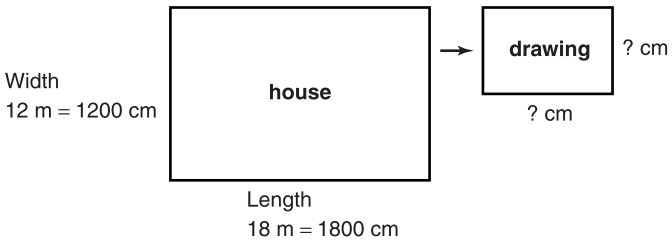
Each 1 cm in the drawing represents 30 cm in the home.

b) Draw a diagram.



The length in the drawing is 16 cm.
Each 1 cm in the drawing represents 30 cm in the bedroom.
So, the length of the bedroom is 16×30 cm $= 480$ cm
$= 4.8$ m

c) Draw a diagram.



1 cm in the drawing represents 30 cm in the house.
Divide each measurement in the house to find the measurement on the drawing.

Length in the drawing $= \frac{1800 \text{ cm}}{30}$
$= 60$ cm

Width in the drawing $= \frac{1200 \text{ cm}}{30}$
$= 40$ cm

The dimensions of the scale drawing are 60 cm by 40 cm.

Example 2

Wallpaper paste can be made by mixing flour and water.
The table shows the volume of water to be mixed with flour to make different quantities of paste.

Water (mL)	16	32	48
Flour (mL)	3	6	9

a) What is the ratio of the volume of water to the volume of flour?

b) How much water is required for 15 mL of flour?

Solution

a) Volume of water:volume of flour = 16:3

b) Use a pattern to continue the table in part a.
For the water, the pattern rule is: Start at 16, then add 16.
The next three terms are 64, 80, 96.
For the flour, the pattern rule is: Start at 3, then add 3.
The next three terms are 12, 15, 18.
Extend the table to include these numbers.

Water (mL)	16	32	48	64	80	96
Flour (mL)	3	6	9	12	15	18

From the table, 80 mL of water are needed for 15 mL of flour.

Practice

Mental Math

Find all the factors of each number.

7, 15, 27, 36, 48, 51

1. This table shows the quantities of concentrate and water needed to make punch.

Orange concentrate (cans)	2	4	6		
Cranberry concentrate (cans)	1	2	3		
Water (cans)	3	6	9		
Punch (L)	1.5	3	4.5		

a) What are the next two numbers in each row of the table?

b) What is the ratio of cranberry concentrate to water?

c) Suppose you use 10 cans of orange concentrate.
How much punch would you get?

d) Suppose you use 12 cans of concentrate in total.
How much punch would you get?

e) How much of each ingredient would you need to make 15 L of punch? Explain your reasoning.

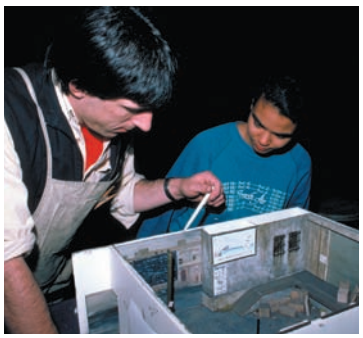
Remember to convert the units so you work with all measurements in the same units.

2. Jenny makes models of Canadian Coast Guard ships.
She uses a scale of 3 cm to 1.5 m.

a) What is the ratio of a length on the model to a length on the ship?

b) The Terry Fox ship is 88 m long. How long is the model?

c) The model of the Otter Bay is 27 cm long.
What is the actual length of the Otter Bay?
Draw diagrams to show your answers.



3. A set designer builds a model of the stage and the different pieces of furniture on it. He uses a scale of 5 cm to 1 m.

a) What is the ratio of a length on the model to a length on the stage?

b) The length of a table is 1.4 m.
What is the length of the model?

c) The height of the model of a lamp is 3 cm.
What is the actual height of the lamp?

d) The height of the model of an ornament is 0.5 cm.
What is the actual height of the ornament?
How did you find out?

Draw diagrams to show your answers.

Math Link

Social Studies

A scale on a provincial map is 1:1 500 000. This means that 1 cm on the map represents 1 500 000 cm on the ground, which is $\frac{1\ 500\ 000}{100}$ m, or 15 000 m on the ground.
And, 15 000 m is $\frac{15\ 000}{1000}$ km, or 15 km.
So, a scale of 1:1 500 000 is 1 cm to 15 km.

4. Janice builds a model using a scale of 2 cm to represent 3 m. Her friend William says she is using a ratio of 2:3. Is William correct? Explain.

5. This table shows the quantities of salt and water needed to make salt solutions.

Water (L)	2	4	6	8
Salt (g)	18	36	54	72

a) How much salt would be needed for 7 L of water?

b) How much water would be needed for 90 g of salt?

6. **Assessment Focus** Aston challenges his father to a 100-m race.
Aston runs 4 m for every 5 m his father runs.

a) Who wins the race? Draw a diagram to show your answer.

b) How far will Aston have run when his father crosses the finish line?

c) Aston asks to race again but wants to be given a head start. How much of a start should Aston's father give him to make it a close race? Explain your answer.

Reflect

Make up your own ratio problem.
Solve your problem. Show your work.

2.5 What Is a Rate?

Focus Investigate rates.

Explore

Work with a partner.
You will need a stopwatch.
One person is the "blinker." The other person is the timekeeper.
The blinker blinks as many times as possible.
Count the number of times the blinker blinks in 20 s.
Reverse roles.
Count the number of times the blinker blinks in 30 s.

➢ Who was the faster blinker?
How do you know?

➢ Estimate how many times each person would blink in 1 h.
What assumptions do you make?
Are these assumptions reasonable?

Reflect & Share

Compare your results with those of another pair of classmates.
How can you decide who is the fastest blinker?

Connect

When we compare two different things, we have a **rate**.
Here are some rates.

- We need 5 sandwiches for every 2 people.
- Oranges are on sale at $1.49 for 12.
- Gina earns $4.75 per hour for baby-sitting.
- There are 500 sheets on one roll of paper towels.

The last two rates above are **unit rates**.
Each rate compares a quantity to 1 unit.

Jamal skipped rope 80 times in 1 min.
We say that Jamal's rate of skipping is 80 skips per minute.
We write this as 80 skips/min.

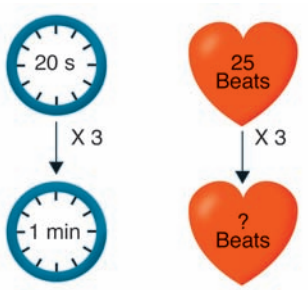
To find unit rates, we can use diagrams, tables, and graphs.

Example 1

The doctor took Marjorie's pulse. He counted 25 beats in 20 s. What was Marjorie's heart rate in beats per minute?

Solution

Draw a diagram.
There are 25 beats in 20 s.
There are 60 s in 1 min.
We multiply 20 s by 3 to get 60 s.
So, multiply the number of beats in 20 s by 3 to get the number of beats per minute.



25 beats $\times$ 3 = 75 beats

Marjorie's heart rate is 75 beats/min.

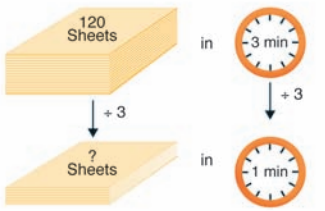
Example 2

A printing press prints 120 sheets in 3 min.

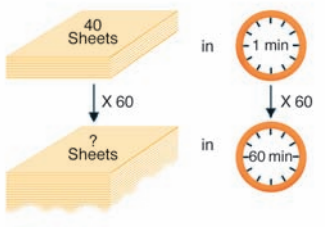
a) Express the printing as a rate.
b) How many sheets are printed in 1 h?
c) How long will it take to print 1000 sheets?

Solution

a) Draw a diagram.
The press prints 120 sheets in 3 min.
So, in 1 min, the press prints:
120 sheets $\div$ 3 = 40 sheets
The rate of printing is 40 sheets/min.



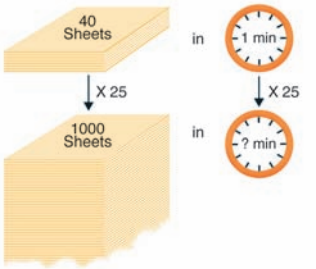
b) In 1 min, the press prints 40 sheets.
One hour is 60 min.
So, in 60 min, the press prints:
60 $\times$ 40 sheets = 2400 sheets
The press prints 2400 sheets in 1 h.



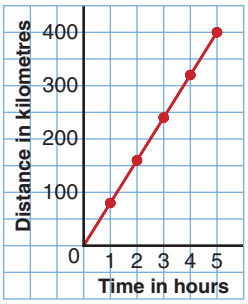
c) *Method 1*
In 1 min, the press prints 40 sheets.
So, in 5 min, the press prints: 5 $\times$ 40 = 200 sheets
Make a table. Every 5 min, 200 more sheets are printed.
Extend the table until you get 1000 sheets.

Time (min)	5	10	15	20	25
Sheets printed	200	400	600	800	1000

Method 2



The press prints 40 sheets in 1 min.
Think: What do we multiply
40 by to get 1000?
Use division: $1000 \div 40 = 25$
So, $40 \times 25 = 1000$
We multiply the time by the same number.
$1 \text{ min} \times 25 = 25 \text{ min}$
The press takes 25 min to print 1000 sheets.



The rate at which a car travels is its **average speed**.
When a car travels at an average speed of 80 km/h, it travels:
80 km in 1 h
160 km in 2 h
240 km in 3 h
320 km in 4 h
400 km in 5 h … and so on
We can show this motion on a graph.
An average speed of 80 km/h is a unit rate.

Practice

1. Express as a unit rate.
a) Morag typed 60 words in one minute.
b) Peter swam 25 m in one minute.
c) Abdu read 20 pages in one hour.

2. Express as a unit rate.
a) June cycled 30 km in 2 h.
b) An elephant travelled 18 km in 30 min.
c) A plane flew 150 km in 15 min.

3. Before running in a 100-m race, Gaalen's heart rate was 70 beats/min. Which do you think is more likely after the race: 60 beats/min or 120 beats/min? Explain.

Mental Math

Find a square root of each number.

49, 25, 81, 16, 64, 144, 4, 121

4. Ribbon costs $1.44 for 3 m.

a) What is the cost per metre?

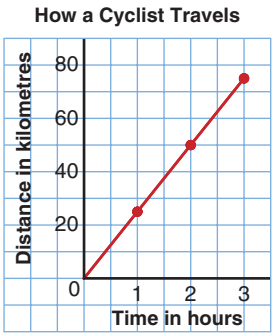
b) How much would 5 m of ribbon cost?

c) How much ribbon could you buy for $12?

5. The graph shows the distance travelled by a cyclist in 3 h.

a) How far did the cyclist travel in 1 h?

b) What is the average speed of the cyclist? How do you know?



In England, the currency is pennies and pounds. There are 100 pennies in one pound, £1.

6. James and Lucinda came to Canada on holiday from England. The rate of exchange for their money was $2.50 Can to £1.

a) How many Canadian dollars would James get for £20?

b) What is the value in English pounds of a gift Lucinda bought for $30 Can?

7. When a person runs a long-distance race, she thinks of the time she takes to run 1 km (min/km), rather than the distance run in 1 min (km/min).
On a training run, Judy took 3 h 20 min to run 25 km.
What was Judy's rate in minutes per kilometre?

8. **Assessment Focus** Scott trained for the marathon.
On day 1, he took 70 min to run 10 km.
On day 10, he took 2 h 40 min to run 20 km.
On day 20, he took 4 h 15 min to run 30 km.

a) What was Scott's running rate, in minutes per kilometre, for each day?

i) Day 1 **ii)** Day 10 **iii)** Day 20

b) What do you think Scott's running rate, in minutes per kilometre, might be for the 44 km of the marathon? How long do you think it will take him? Explain.

Reflect

Look through newspapers and magazines to find three different examples of rates. Explain how the rates are used.

Organizing a Math Notebook

1. You must include the date for each new note taken or activity performed.

2. You must include a title for each new note or activity.

3. All dates and titles must be underlined with a ruler.

4. All tables must be neat.
Use a ruler to draw a table.

For example:

Section 2.5, question 8.

April 13, 2005

Day	*Time (min)*	*Distance (km)*	*Rate (min/km)*
1			

5. Your daily work must be legible, complete, and well-organized.

Reading and Writing in Math

6 You must make corrections where necessary.

7 To organize a math problem:

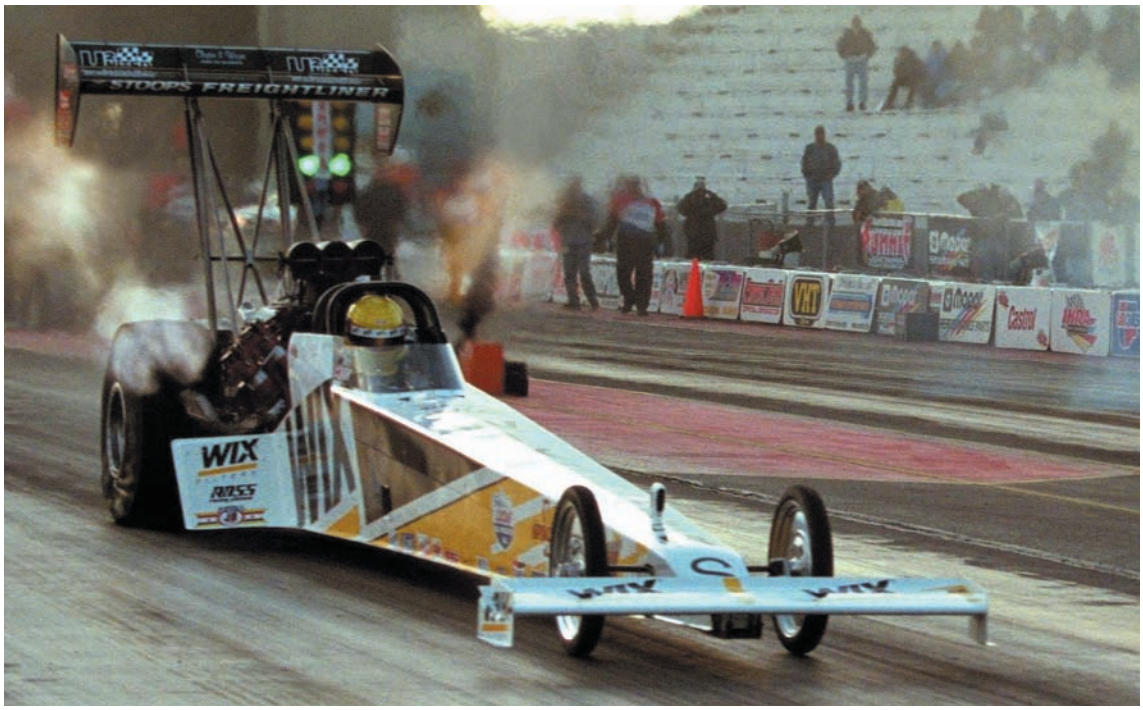
- Restate the problem in your own words.
  For example, "This problem is about…"
- Think about a strategy you will use and tell about it.
  For example, "The strategy I will use is…_____ because…"
- Solve the problem. Show all your work.
- State the answer to your problem, and explain how you know it is reasonable and correct.
  For example, "I found the answer to be…
  I know my answer is reasonable because…
  I know my answer is correct because…"
- Extend your thinking. Make up a similar problem by asking "What if?" questions.
  For example, "What if all the numbers in this question were doubled? How would the answer change?"

The World of Work

Race Engineer

The equivalent of the 100-m sprint in the world of cars is the $\frac{1}{4}$-mile drag race. One-quarter of one mile is about 400 m. The race engineers (men and women who help design, build, and tune the cars) are an important part of the racing team. Theirs is a world filled with gear ratios, cylinder compression ratios, fuel mixture ratios, acceleration, and speed. It's a constant cycle of theory, real-world application, testing, and evaluating. And, sometimes, all these take place during a few hours between race runs! Helping the dragster team driver get to the finish line $\frac{1}{1000}$ th of a second faster than a previous run could mean the difference between winning and losing the race.

In October, 2003, a top fuel dragster recorded a new "fastest time" of 4.441 s for the $\frac{1}{4}$-mile drag race. But another car and driver continued to hold the record for the fastest recorded speed achieved during a race—an incredible 536 km/h! Why do you suppose the second car doesn't hold the record for fastest time?



Unit Review

Review any lesson with etext online tutorial

What Do I Need to Know?

- A ratio is a comparison of quantities.
  For example, 3 dogs to 7 cats is 3:7.

- An equivalent ratio can be formed by multiplying or dividing the terms of a ratio by the same number.

  For example:

5:8	and	36:30
$= (5 \times 3):(8 \times 3)$		$= (36 \div 6):(30 \div 6)$
$= 15:24$		$= 6:5$
5:8 and 15:24 are equivalent ratios.		36:30 and 6:5 are equivalent ratios.

- Two ratios can be compared when the second terms are the same.
  For example, Scott's scoring record was 16:5.
  Brittany's scoring record was 10:3.
  To find who had the better record, use equivalent ratios to make the second term of each ratio 15.

Scott	**Brittany**
16:5	10:3
$= (16 \times 3):(5 \times 3)$	$= (10 \times 5):(3 \times 5)$
$= 48:15$	$= 50:15$

Brittany had the better record.

- A rate is a comparison of two quantities with different units.
  For example:
  Heart rate is measured in beats per minute (beats/min).
  Average speed is measured in kilometres per hour (km/h).
  Fuel consumption of an aircraft is measured in litres per hour (L/h).

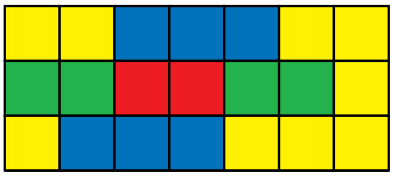
What Should I Be Able to Do?

For extra practice, go to page 439.

LESSON

2.1 **1.** On a school trip, there are 9 boys, 10 girls, and 4 adults.
Write each ratio.
a) girls to boys
b) boys and girls to adults
c) adults to boys and girls

2.2 **2.** Write four ratios to describe the coloured squares.



3. Explain two different ways to get ratios equivalent to 25:10.

4. Jake has to draw a flag with a length to width ratio of 3 to 2. What is the largest flag Jake can draw on a 25-cm by 20-cm sheet of paper?

2.3 **5.** The ratio of computers to students in Ms. Beveridge's class is 2:3. The ratio of computers to students in Mr. Walker's class is 3:5. Each class has the same number of students.
Which room has more computers? Explain.

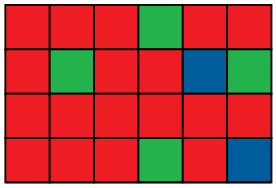
2.4 **6.** Ali builds model planes. He uses a scale of 8 cm to represent 1.8 m.
a) What is the ratio of a length on the model to the actual length on the plane?
b) A plane has a length of 72 m. What is the length of the model?
c) A model has a length of 60 cm. What is the actual length of the plane?
Draw pictures to show your answers.

7. Red and white paint is mixed in the ratio of 3 to 2.
a) How many cans of red paint would be needed with 6 cans of white paint?
b) How many cans of each colour are needed to make 20 cans of mixture?

2.5 **8.** Express as a unit rate.
a) A bus travelled 120 km in 3 h.
b) An athlete ran 1500 m in 6 min.
c) A student earned $16 for 2 h work.

9. A lion can run 550 m in 25 s.
A zebra can run 270 m in 15 s.
a) Which animal is faster?
b) What is the ratio of their average speeds?

Practice Test



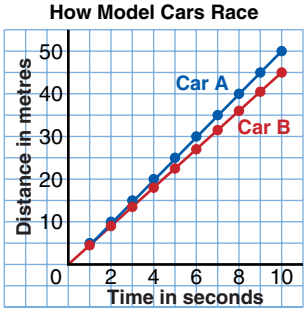
1. **a)** Write each ratio in simplest form.
- **i)** red squares to blue squares
- **ii)** blue squares to green squares
- **iii)** red squares and blue squares to total numbers of squares

b) Suppose the grid is increased to a rectangle measuring 9 units by 4 units. The ratios of the colours remain the same. How many red squares will there be in the new rectangle?

2. In the league, the Leos play 8 games. Their win to loss ratio is 5:3. The Tigers play 11 games. Their win to loss ratio is 7:4.
- **a)** Which team has the better record? Explain.
- **b)** Suppose the Leos win their next game and the Tigers lose theirs. Which team would have the better record? Explain.

3. Hessa is building a scale model of a park.
She uses a scale of 12 cm to represent 1.5 m.
- **a)** The actual length of the bridge is 20 m. What is the length of the model bridge?
- **b)** In the model, the height of the climbing frame is 10 cm. What is the actual height of the frame?

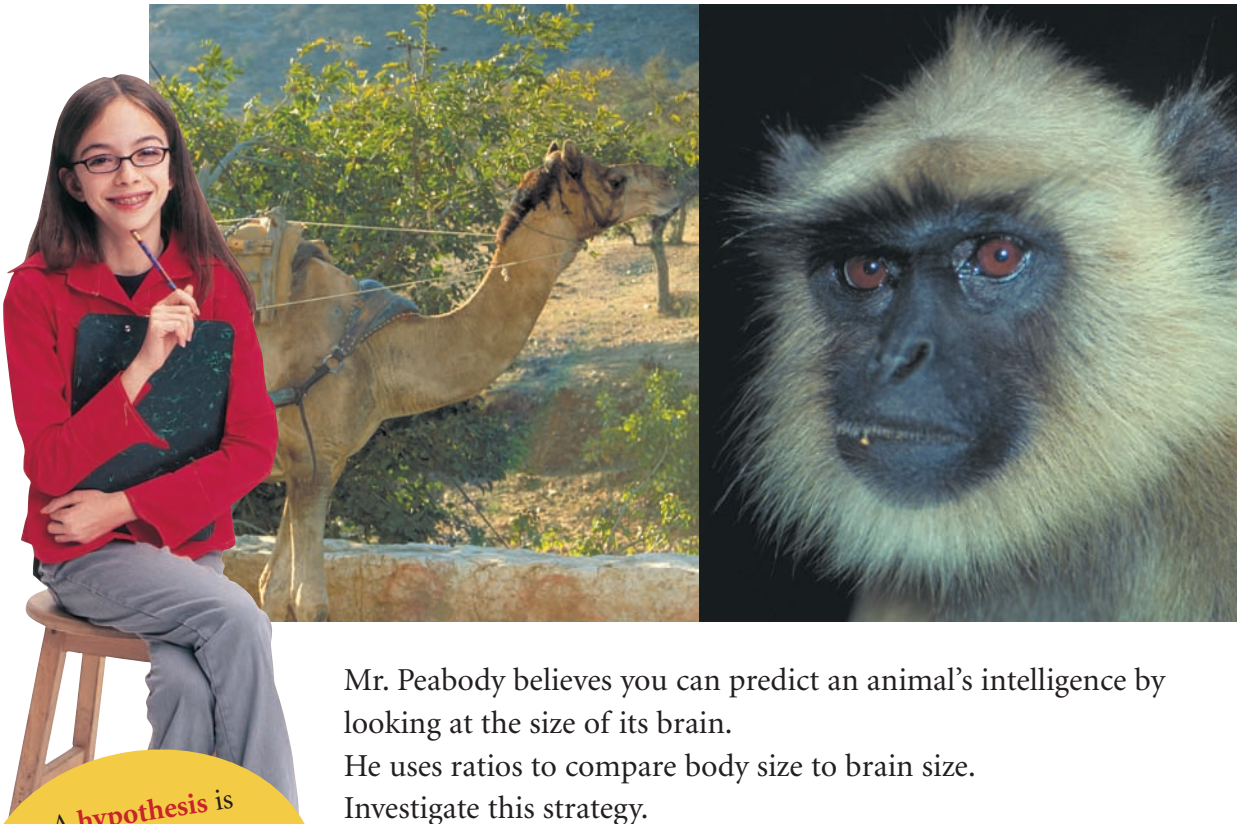
4. Look at this graph.
- **a)** What is the speed of each car?
- **b)** How far apart are the cars after 4 s?



5. Trevor's mark on a math test was 10 out of 15.
Anne's mark on another test was 15 out of 20.
Trevor said, "Each of us got 5 wrong. So, our marks are equal."
Do you agree? Give reasons for your answer.

Unit Problem

Who's the Smartest?



A **hypothesis** is something that seems likely to be true. It needs to be tested and proved or disproved.

The mass of a human brain is about the same mass as your math textbook.

Mr. Peabody believes you can predict an animal's intelligence by looking at the size of its brain.
He uses ratios to compare body size to brain size.
Investigate this strategy.

Analyse the data below to test Mr. Peabody's **hypothesis**.

Species	Comparing Mass (g)		Comparing Length (cm)	
	Body	Brain	Body	Brain
Human	56 000	1400	150	15
Monkey	7 000	100	30	5
Camel	520 000	650	200	15

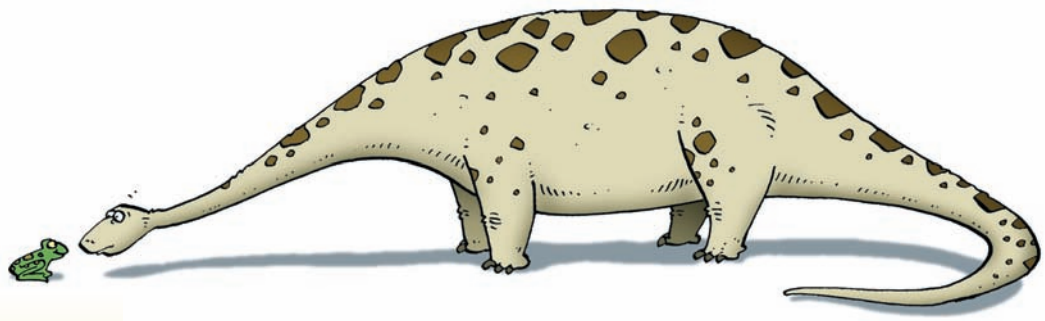
1. Compare masses: find the ratio of body mass to brain mass.
- How does a human compare to a monkey?
- How does a human compare to a camel?
- How does a camel compare to a monkey?

According to Mr. Peabody's hypothesis, which animal is smartest? Explain your reasoning.

2. Compare lengths: find the ratio of body length to brain length.

- How does a human compare to a monkey?
- How does a human compare to a camel?
- How does a camel compare to a monkey?

According to Mr. Peabody's hypothesis, which animal is smartest? Explain your reasoning.



Check List

Your work should show:

- ✓ how you calculated each ratio
- ✓ your reasoning for which animal is the smartest
- ✓ your conclusions about the hypothesis
- ✓ the correct use of mathematical language

3. The dinosaur, diplodocus, lived about 150 million years ago.
The brain of a diplodocus was about 9 cm long.
The body of a diplodocus was about 27 m long.

- How long would a human's brain be if it had the same brain length to body length ratio as a diplodocus?

A frog's brain is about 2 cm long.
A frog's length is about 10 cm long

- How long would a human's brain be if it had the same brain length to body length ratio as a frog?

4. Review your results.
Write a short letter to Mr. Peabody telling him whether you agree with his hypothesis, and why.
Use mathematical language to support your opinion.

Reflect on the Unit

How is a rate like a ratio?
How is it different?
Use examples in your explanation.