

UNIT

1

Patterns in Whole Numbers

There are many patterns you can see in nature. You can use numbers to describe many of these patterns.

At the end of this unit, you will investigate a famous set of numbers, the Fibonacci Numbers. You will explore these numbers, and find how they can be used to describe the breeding of animals, such as rabbits.

Think about the different patterns you learned in earlier grades.

Give an example of each type of pattern.

What You'll Learn

- Use mental math, paper and pencil, calculators, and estimation to solve problems.
- Justify your choice of method for calculations.
- Find factors and multiples of numbers.
- Find the greatest common factor and lowest common multiple of two numbers.
- Identify prime and composite numbers.
- Use exponents to represent repeated multiplication.
- Identify and extend number patterns.
- Choose and justify strategies for solving problems.

Why It's Important

- You need to know an appropriate and efficient method for calculations.
- You need to be able to find common denominators for fraction calculations.





Key Words

- factor
- prime number
- composite number
- greatest common factor (GCF)
- multiple
- lowest common multiple (LCM)
- square number
- square root
- exponent form
- base
- exponent
- power
- cube number
- perfect square
- perfect cube

Skills You'll Need

Rounding

The place-value chart below shows the number 1 234 567.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1	2	3	4	5	6	7

To round:

Look at the digit to the right of the place to which you are rounding.

Is this digit 5 or greater?

If it is, add 1 to the place digit.

If it is not, leave the place digit as it is.

Change all the digits to the right of the place digit to 0.

Example 1

a) Round 425 to the nearest ten.

b) Round to the nearest thousand.

i) 2471

ii) 13 999

Solution

a)

Hundreds	Tens	Ones
4	2	5

This number is 5.

So, add 1 ten to this number to get 3 tens.
Then, replace 5 with 0.

425 rounded to the nearest ten is 430.

b) i)

Thousands	Hundreds	Tens	Ones
2	4	7	1

This number is less than 5.

So, this number does not change.
Replace each number to the right of 2 with 0.

2471 rounded to the nearest thousand is 2000.

ii)

Ten thousands	Thousands	Hundreds	Tens	Ones
1	3	9	9	9



This number is greater than 5.

So, add 1 thousand to this number to get 4 thousands.
Replace each number to the right of 4 with 0.

13 999 rounded to the nearest thousand is 14 000.

✓ Check

1. Round to the nearest ten.

a) 36

b) 42

c) 75

d) 361

2. Round to the nearest hundred.

a) 311

b) 789

c) 625

d) 2356

Multiplying by 10, 100, 1000

To multiply a whole number:

– by 10, write 0 after the number.

– by 100, write 00 after the number.

– by 1000, write 000 after the number.

Example 2

Multiply.

a) 32×10

b) 478×100

c) 51×1000

d) 32×20

e) 47×300

Solution

a) $32 \times 10 = 320$

b) $478 \times 100 = 47\,800$

c) $51 \times 1000 = 51\,000$

d) $32 \times 20 = 32 \times 2 \times 10$

e) $47 \times 300 = 47 \times 3 \times 100$

$= 64 \times 10$

$= 141 \times 100$

$= 640$

$= 14\,100$

✓ Check

3. Find.
- a) 3×10 b) 1000×5 c) 131×10 d) 100×63
4. Use a place-value chart. Explain why we can write zeros after a number when we multiply by 10, 100, or 1000.
5. Multiply.
- a) 50×72 b) 18×600 c) 4000×33

Mental Math

Example 3

Use mental math.

- a) 53×6 b) $308 + 56 - 6$ c) $197 + 452$

Solution

- a) 53×6 b) $308 + 56 - 6$

Think:

$$\begin{aligned} & 50 \times 6 + 3 \times 6 \\ & = 300 + 18 \\ & = 318 \end{aligned}$$

Think:

$$\begin{aligned} & 56 - 6 = 50 \\ & \text{Then: } 308 + 50 = 358 \end{aligned}$$

- c) $197 + 452$

Think:

$$\begin{aligned} & 197 = 200 - 3 \\ & \text{Then: } 200 + 452 - 3 = 652 - 3 \quad \text{Count back to subtract.} \\ & = 649 \end{aligned}$$

✓ Check

6. Use mental math.
- a) $4 + 17$ b) $9 + 8$ c) $12 + 6$
d) $20 + 6$ e) $40 + 30$ f) $17 - 2$
g) $22 - 4$ h) $70 - 20$ i) $20 - 15$
7. Use mental math. Explain your strategy.
- a) $28 + 13 + 12$ b) $2 \times 29 \times 5$ c) $98 + 327$
d) $4 \times 981 \times 25$ e) 99×21 f) 62×8

Divisibility Rules

A number is divisible by:

- 2 if the number is even
- 3 if the sum of its digits is divisible by 3
- 4 if the number represented by the last 2 digits is divisible by 4
- 5 if the last digit is 0 or 5
- 6 if the number is divisible by 2 and 3
- 8 if the number represented by the last 3 digits is divisible by 8
- 9 if the sum of the digits is divisible by 9
- 10 if the last digit is 0

Example 4

Which of these numbers is 1792 divisible by?

- a) 2 b) 3 c) 4 d) 5 e) 6

Solution

- a) 1792 is divisible by 2 because 1792 is an even number.
- b) $1 + 7 + 9 + 2 = 19$
19 is not divisible by 3, so 1792 is not divisible by 3.
- c) The last 2 digits are 92.
 $92 \div 4 = 23$
Since the last 2 digits are divisible by 4, 1792 is divisible by 4.
- d) 1792 is not divisible by 5 because the last digit is not 0 or 5.
- e) Since 1792 is not divisible by 3, 1792 is also not divisible by 6.

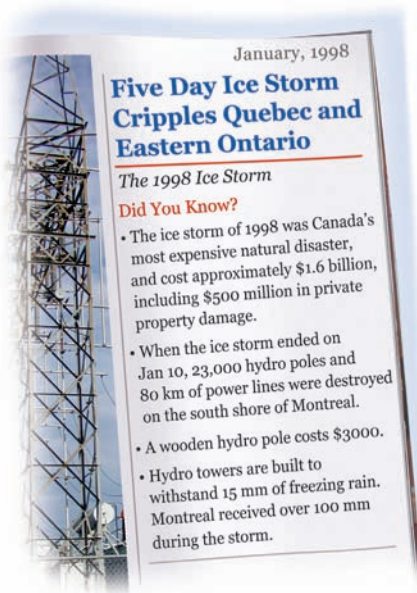
Check

8. Which numbers are divisible by 3?
a) 490 b) 492 c) 12 345
9. Write 4 other numbers greater than 400 that are divisible by 3.
10. Which numbers are divisible by 6?
a) 870 b) 232 c) 681
11. Which numbers from 1 to 10 is:
a) 660 divisible by? b) 1001 divisible by?

We use numbers to understand and describe our world.



Explore



Work on your own.

Read the articles above and at the left.

- Which numbers do you think are exact?
Which numbers are estimates? Explain your thinking.
- Use the numbers in the articles.
Write a problem you would solve each way:
 - using mental math
 - by estimating
 - using pencil and paper
 - using a calculator
- Solve your problem.
- Trade problems with a classmate.
Solve your classmate's problem.

Reflect & Share

Compare the strategies you used to solve the problems.

- Explain why some strategies work while others may not.
- Is one strategy more effective? Why?

- When the numbers are easy to handle, use mental math.
- When the problem has too many steps, use a paper and pencil.
- When an approximate answer is appropriate and to check reasonableness, estimate.
- When a more accurate answer is needed and the numbers are large, use a calculator.

Example

The population of Canada was 30 750 000 in July 2000. Statistics Canada (Stats Can) data show that there were 6367 telephones per 10 000 people in that year.

- About how many telephones were there in Canada in 2000?
- Find the exact number of telephones in Canada in 2000.
- How did Stats Can know there were 6367 phones per 10 000 people? Explain how this answer affects the answer to part b.

Solution

- Estimate.

Round 30 750 000 to the nearest ten million: 30 000 000

Round 6367 to the nearest thousand: 6000

10 000 people use about 6000 phones.

10 000 000 people use about 6000×1000 , or 6 000 000 phones.

30 000 000 people use about $6\,000\,000 \times 3$, or

18 000 000 phones.

There were about 18 million phones in Canada in 2000.

- Find how many groups of 10 000 people there are in 30 750 000.

That is, $30\,750\,000 \div 10\,000 = 3075$

For each group of 10 000 people, there were 6367 phones.

The number of phones: $3075 \times 6367 = 19\,578\,525$

So, there were 19 578 525 phones in Canada in 2000.

- Stats Can conducted a survey of about 40 000 people.

It asked how many phones each person had.

Stats Can then calculated how many phones per 10 000 people.

To find out exactly how many phones there are in Canada, Stats Can would have to survey the entire population. This is impractical. So, the number of phones in part b is an estimate.



Use a calculator.

Practice

- Solve without a calculator.
 - $72 + 43$
 - $123 + 85$
 - $672 + 189$
 - $97 - 24$
 - $195 - 71$
 - $821 - 485$
 - 65×100
 - 14×75
 - 83×25
 - $780 \div 10$
 - $724 \div 4$
 - $245 \div 7$
- Use pencil and paper to find each answer.
 - $6825 + 127$
 - $7928 - 815$
 - $3614 - 278$
 - 138×21
 - $651 \div 21$
 - $6045 \div 15$
- Estimate each answer. Explain the strategy you used each time.
 - $103 + 89$
 - $123 - 19$
 - 72×9
 - $418 \div 71$

Mental Math

Multiply.

$3.6 \times 1000 \quad 3.6 \times 100$

$3.6 \times 10 \quad 3.6 \times 1$

$3.6 \times 0.1 \quad 3.6 \times 0.01$

What patterns do you see in the questions and the answers?

Questions 4 to 7 pose problems about the 1997 Red River Flood in Manitoba. Use mental math, estimation, pencil and paper, or a calculator. Justify your strategy.

- The 1997 Red River Flood caused over \$815 036 000 in damages.
 - Write this amount in words.
 - How close to \$1 billion were the damages?
- Pauline Thiessen and fellow volunteers made an average of 10 000 sandwiches every day for 2 weeks to feed the flood relief workers. How many sandwiches did they make?
- Winnipeg used 6.5 million sandbags to hold back the flood. Each sandbag was about 10 cm thick. About how high would a stack of 6.5 million sandbags be in centimetres? Metres? Kilometres?
- To help with the flood relief, Joe Morena of St. Viateur Bagels in Montreal trucked 300 dozen of his famous bagels to Manitoba.
 - How many bagels did he send?
 - Joe sells bagels for \$4.80 a dozen.
What was the value of his donation?
- The Monarch butterfly migrates from Toronto to El Rosario, Mexico. This is a distance of 3300 km.
A monarch butterfly can fly at an average speed of 15 km/h.
How long does the migration flight take?



9. Estimate each answer. Is each estimate high or low?
How do you know?
a) $583 + 702$ b) $3815 - 576$ c) $821 \div 193$ d) $695 \div 310$

For questions 10 and 11: Make up a problem using the given data.
Have a classmate solve your problems.

10. Sunil earns \$7 per hour. He works 4 h per day during the week and 6 h per day on the weekends.
11. In October 1954, Hurricane Hazel blew through Toronto, Ontario. Winds reached 124 km/h, 111 mm of rain fell in 12 h, and over 210 mm of rain fell over 2 days.



12. **Assessment Focus** The table shows the populations of some Canadian provinces in 1999.

Province	Population
NF and Labrador	541 000
PEI	138 000
Nova Scotia	939 800
New Brunswick	755 000
Ontario	11 513 800

The **mean** of a set of numbers is the sum of the numbers divided by how many numbers there are.

- a) Do you think these numbers are exact? Explain.
b) Find the total population of the 4 Atlantic provinces.
c) Find the mean population of the Atlantic provinces.
d) Approximately how many times as many people are in Ontario as are in the Atlantic provinces?
e) Make up your own problem about these data. Solve it.

Take It Further

13. Find 2 whole numbers that:
a) have a sum of 10 and a product of 24
b) have a difference of 4 and a product of 77
c) have a sum of 77 when added to 3
Which of parts a to c have more than one answer? Explain.

Reflect

Write an example of a problem you would solve:

- by estimation
- by using a calculator

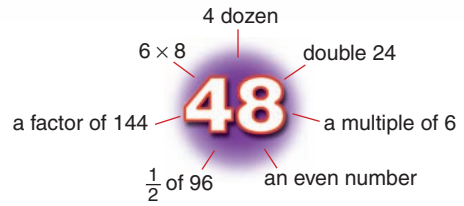
Justify your choice.

1.2

Factors and Multiples

Focus Generate factors and multiples of given numbers.

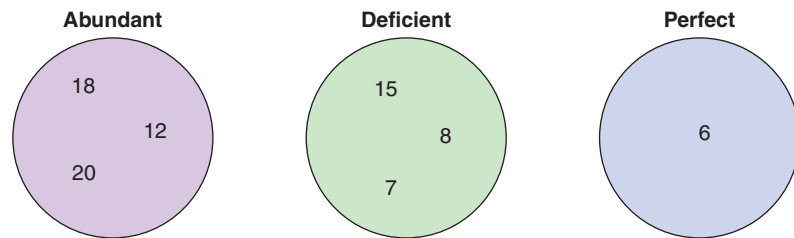
Just as numbers can describe our world, we can use numbers to describe other numbers. We can describe a number by its factors, by the number of its factors, and by the sum of its factors.



Explore

Work with a partner.

- Analyse the numbers in the circles.



Use a table to record the factors of each number.
Cross out the number itself.
Find the sum of the remaining factors.

Number	Factors	Sum of Factors

- Look for patterns among your results.
Why do you think a number is called “Abundant,” “Deficient,” or “Perfect”?
- Copy the 3 circles. Based on your ideas, place each number from 2 to 30 in the appropriate circle. What do you notice?
- Use your observations to predict where 36 and 56 belong.
Check your predictions.

Reflect & Share

Share your results with another pair of students.
What relationships did you see?
How did you describe each type of number?

➤ Recall that a factor is a number that divides exactly into another number.

For example, 1, 2, 3, and 6 are the factors of 6.

Each number divides into 6 with no remainder.

A **prime number** has only 2 factors, itself and 1.

2, 3, and 5 are prime numbers.

All prime numbers are deficient.

A **composite number** has more than 2 factors.

8 is a composite number because its factors are 1, 2, 4, and 8.

Composite numbers can be deficient, abundant, or perfect.

1 has only one factor, so 1 is neither prime nor composite.

When we find the factors that are the same for 2 numbers, we find **common factors**.

Example 1

Show the factors of 12 and 30 in a Venn diagram.

What is the **greatest common factor (GCF)** of 12 and 30?

Solution

Use the divisibility rules.

Find pairs of numbers that divide into 12 exactly.

$$12 \div 1 = 12 \quad \text{1 and 12 are factors.}$$

$$12 \div 2 = 6 \quad \text{2 and 6 are factors.}$$

$$12 \div 3 = 4 \quad \text{3 and 4 are factors.}$$

Stop at 3 because the next number, 4, is already a factor.

The factors of 12 are 1, 2, 3, 4, 6, and 12.

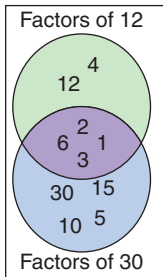
Find the factors of 30.

$$30 \div 1 = 30 \quad \text{1 and 30 are factors.}$$

$$30 \div 2 = 15 \quad \text{2 and 15 are factors.}$$

$$30 \div 3 = 10 \quad \text{3 and 10 are factors.}$$

$$30 \div 5 = 6 \quad \text{5 and 6 are factors.}$$



Mark the factors on the Venn diagram.

Place the common factors in the overlapping region.

The GCF of 12 and 30 is 6.

- The **multiples** of a number are found by multiplying the number by 1, by 2, by 3, by 4, and so on, or by skip counting.

When we find multiples that are the same for 2 numbers, we find **common multiples**.

We can use a 100 chart to find multiples and common multiples.

Example 2

- a) Use a 100 chart to find the common multiples of 12 and 21.
 b) Find the **lowest common multiple (LCM)** of 12 and 21.

Solution

a)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

The multiples of 12 are blue.
 The multiples of 21 are shaded green.
 The common multiple is blue and shaded green.

- b) Multiples of 12 are 12, 24, 36, 48, 60, 72, **84**, 96, ...
 Multiples of 21 are 21, 42, 63, **84**, ...
 The LCM is 84.

Practice

- List 4 multiples of each number.
 a) 5 b) 7 c) 8
- Find the factors of each number. Explain how you did it.
 a) 18 b) 20 c) 28 d) 36 e) 37 f) 45
- Find the factors of each number.
 a) 50 b) 51 c) 67 d) 75 e) 84 f) 120

Number Strategies

Three CDs cost \$12.99, \$8.18, and \$7.88 (tax included).

Approximately how much change should there be from \$40.00?

4. Is each number prime or composite? How do you know?
a) 18 b) 13 c) 9 d) 19 e) 61 f) 2
5. Find the GCF of each pair of numbers.
Which strategy did you use?
a) 10, 5 b) 12, 8 c) 15, 25 d) 9, 12 e) 18, 15
6. Use a 100 chart. Find the LCM of each pair of numbers.
a) 3, 4 b) 2, 5 c) 12, 18 d) 10, 25 e) 27, 18
7. Can a pair of numbers have:
a) more than one common multiple?
b) more than one common factor?
Use a diagram to explain your thinking.
8. Julia and Sandhu bought packages of granola bars.
a) Julia had 15 bars in total. Sandhu had 12 bars in total.
How many bars could there be in one package?
b) What if Julia had 24 bars and Sandu had 18 bars?
How many bars could there be in one package?
Draw a diagram to explain your thinking.
9. **Assessment Focus** Kevin, Alison, and Fred work part-time. Kevin works every second day. Alison works every third day. Fred works every fourth day. Today they all worked together. When will they work together again?
Explain how you know.
10. The numbers 4 and 16 could be called “near-perfect”.
Why do you think this name is appropriate?
Find another example of a near-perfect number.
What strategy did you use?



Reflect

What is the difference between a factor and a multiple?
Is a factor ever a multiple? Is a multiple ever a factor?
Use diagrams, pictures, or a 100 chart to explain.



The Factor Game

YOU WILL NEED

One gameboard;
2 coloured markers

NUMBER OF PLAYERS

2

GOAL OF THE GAME

To circle factors of
a number

HOW TO PLAY THE GAME:

1. Roll a number cube. The person with the greater number goes first.
2. Player A circles a number on the game board and scores that number. Player B uses a different colour to circle all the factors of that number not already circled. She scores the sum of the numbers she circles.

For example, suppose Player A circles 18.

Player B circles 1, 2, 3, 6, and 9 (18 is already circled) to score $1 + 2 + 3 + 6 + 9 = 21$ points

3. Player B circles a new number. Player A circles all the factors of that number not already circled. Record the scores.

4. Continue playing. If a player chooses a number with no factors left to circle, the number is crossed out. The player loses her or his turn, and scores no points.



1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64

What is the best first move? What is the worst first move? Why? How does this game involve factors, multiples, and prime numbers?

For example, if player A circled 16, but 1, 2, 4, and 8 have already been circled, he would lose his turn and score no points.

5. The game continues until all numbers have been circled or crossed out. The player with the higher score wins.

We can get 49 by multiplying the whole number, 7, by itself.

$$49 = 7 \times 7$$

We write: $7 \times 7 = 7^2$

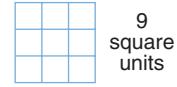
We say: 7 squared

- One way to model a square number is to draw a square.

This square has area 9 square units.

The side length is $\sqrt{9}$, or 3 units.

We say: A **square root** of 9 is 3.



Other inverse operations are addition and subtraction, multiplication and division

- When we multiply a number by itself, we square the number. Squaring a number and taking its square root are inverse operations. That is, they undo each other.

$$7 \times 7 = 49$$

$$\text{so, } 7^2 = 49$$

$$\sqrt{49} = \sqrt{7^2}$$

$$= 7$$

Example 1

Find the square of each number.

a) 5

b) 15

c) 32

Solution

$$\begin{aligned} \text{a) } 5^2 &= 5 \times 5 \\ &= 25 \end{aligned}$$

$$\begin{aligned} \text{b) } 15^2 &= 15 \times 15 \\ &= 225 \end{aligned}$$

$$\begin{aligned} \text{c) } 32^2 &= 32 \times 32 \\ &= 1024 \end{aligned}$$

Use a calculator.

Example 2

Draw a diagram to find a square root of each number.

a) 16

b) 36

Solution

- a) On grid paper, draw a square with area 16 square units.

The side length of the square is 4 units.

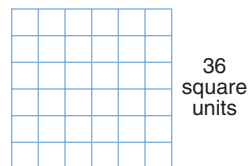
$$\text{So, } \sqrt{16} = 4$$



- b) On grid paper, draw a square with area 36 square units.

The side length of the square is 6 units.

$$\text{So, } \sqrt{36} = 6$$



Practice

Number Strategies

Chris had saved \$145.98 by June. He spent \$2.25 in July, \$4.50 in August, and \$9.00 in September. If Chris' spending pattern continues, when will he have less than \$5.00?

- Find.
a) 8^2 b) 16^2 c) 1^2 d) 29^2
- Find the square of each number.
a) 4 b) 17 c) 13 d) 52
- a) Find the square of each number.
i) 1 ii) 10 iii) 100 iv) 1000
b) Use the results of part a. Predict the square of each number.
i) 10 000 ii) 1 000 000
- Use grid paper. Find a square root of each number.
a) 16 b) 4 c) 900 d) 144
- Calculate the side length of a square with each area.
a) 100 m^2 b) 64 cm^2 c) 81 m^2
- Order from least to greatest.
a) $\sqrt{36}$, 36, 4, $\sqrt{9}$ b) $\sqrt{400}$, $\sqrt{100}$, 19, 15
- Which whole numbers have squares between 50 and 200?
- Assessment Focus** Which whole numbers have square roots between 1 and 20? How do you know?
- A large square room has an area of 144 m^2 .
a) Find the length of a side of the room.
b) How much baseboard is needed to go around the room?
c) Each piece of baseboard is 2.5 m long. How many pieces of baseboard are needed?
- A garden has an area of 400 m^2 .
The garden is divided into 16 congruent square plots.
What is the side length of each plot?



Reflect

How can you find the perimeter of a square when you know its area? Use an example to explain.

Mid-Unit Review

LESSON

- 1.1** 1. The table shows the most common surnames for adults in the United Kingdom.

Surname	Number
Smith	538 369
Jones	402 489
Williams	279 150

- a) Approximately how many adults have one of these three names?
To which place value did you estimate? Explain your choice.
- b) Exactly how many more Smiths are there than Jones? Explain.
- c) Write your own problem about these data. Solve your problem. Justify your strategy.

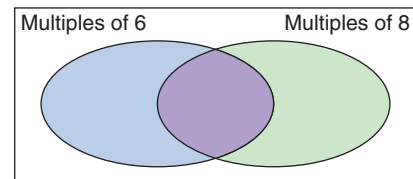
2. In one week, Joe worked 23 h cutting grass. He was paid \$9/h. From this money, Joe bought 5 tickets for a football game, at \$15 per ticket, and 2 DVDs for \$21 each, including tax. How much money did Joe have left?



- 1.2** 3. Find all the factors of each number.
- a) 35 b) 24

4. Find 2 factors of each number.
- a) 6 b) 10
c) 14 d) 15
e) 9 f) 21

5. Organize the first 8 multiples of 6 and 8 in a Venn diagram.



6. For the numbers 15 and 6, find:
- a) the GCF b) the LCM
7. a) Why is 7 a prime number?
b) Why is 8 not a prime number?

- 1.3** 8. Can three consecutive whole numbers all be primes? Justify your answer.

9. A square patio has an area of 81 m^2 . How long is each side?
10. Find.
- a) $\sqrt{49}$ b) 8^2
c) $\sqrt{100}$ d) the square of 9
11. Find two squares with a sum of 100.
12. Write 100 as a square number and as a square root of a number.
13. Explain why:
 $\sqrt{1} = 1$

Focus Use exponents to represent repeated multiplication.

Explore

Work with a small group.

You will need 65 interlocking cubes.

The edge length of each cube is 1 unit.

The volume of each cube is 1 cubic unit.

- How many different ways can you make a larger cube?
- What is the volume of each larger cube you make?
What is its edge length?
- Use factors to write the volume of each cube.
- Record your results in a table.



Number of Cubes	Volume (cubic units)	Edge Length (units)	Volume As a Product
1	1	1	$1 \times 1 \times 1$

Reflect & Share

Observe how the volume grows. Describe the growth using pictures or numbers. What other patterns do you see in the table?

Use these patterns to help you write the volumes of the next 3 cubes in the pattern.

Connect



When numbers are repeated in multiplication, we can write them in **exponent form**.

For example, we can write $2 \times 2 \times 2 \times 2$ as 2^4 .

2 is the **base**.

4 is the **exponent**.

2^4 is the **power**.

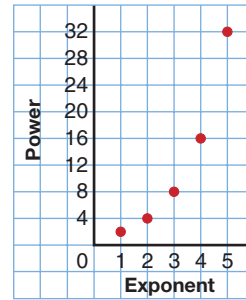


We say: 2 to the power of 4, or
2 to the 4th

2^4 is a power of 2.

If we graph the power against the exponent, we see how quickly the power gets very large.

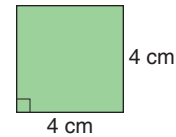
Exponent	Power
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
4	$2^4 = 16$
5	$2^5 = 32$



Square numbers and cube numbers are special powers.

- A power with exponent 2 is a **square number**.
The area of a square is side length \times side length.
This square has side length 4 cm.

$$\begin{aligned} \text{Area} &= 4 \text{ cm} \times 4 \text{ cm} \\ &= 16 \text{ cm}^2 \end{aligned}$$



Here are 3 ways to write 16:

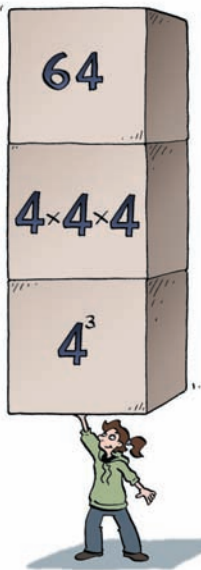
Standard form: 16

Expanded form: 4×4

Exponent form: 4^2

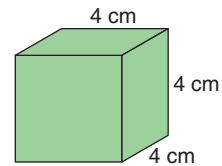
4^2 is a power of 4.

16 is called a **perfect square**.



- A power with exponent 3 is a **cube number**.
The volume of a cube is edge length \times edge length \times edge length.
This cube has edge length 4 cm.

$$\begin{aligned} \text{Volume} &= 4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm} \\ &= 64 \text{ cm}^3 \end{aligned}$$



Here are 3 ways to write 64:

Standard form: 64

Expanded form: $4 \times 4 \times 4$

Exponent form: 4^3

4^3 is a power of 4.

64 is called a **perfect cube**.

Example 1

Write in exponent form.

a) 6×6 b) $5 \times 5 \times 5$ c) 32

Solution

a) $6 \times 6 = 6^2$ b) $5 \times 5 \times 5 = 5^3$

c) $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$

Example 2

Write in expanded form and standard form.

a) 3^5 b) 7^4

Solution

a) 3^5 b) 7^4
 $= 3 \times 3 \times 3 \times 3 \times 3$ $= 7 \times 7 \times 7 \times 7$
 $= 243$ $= 2401$

A calculator can be used to simplify a power such as 3^5 .

For a scientific calculator, the keystrokes are:

$\boxed{3}$ $\boxed{\wedge}$ $\boxed{5}$ $\boxed{\text{ENTER}}$ or $\boxed{3}$ $\boxed{y^x}$ $\boxed{5}$ $\boxed{\text{ENTER}}$ to display 243

For a non-scientific calculator, use repeated multiplication.

The keystrokes are:

$\boxed{3}$ $\boxed{\times}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ to display 243

Practice

1. Write the base of each power.

a) 2^4 b) 3^2 c) 7^3 d) 10^5 e) 6^9 f) 8^3

2. Write the exponent of each power.

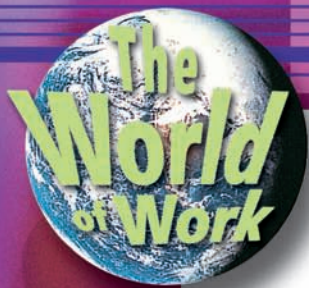
a) 2^5 b) 3^2 c) 7^1 d) 9^5 e) 8^{10} f) 10^4

3. Write in expanded form.

a) 2^4 b) 10^3 c) 6^5 d) 4^2 e) 2^1 f) 5^4

4. Write in exponent form.

a) $3 \times 3 \times 3 \times 3$ b) $2 \times 2 \times 2$ c) $5 \times 5 \times 5 \times 5 \times 5 \times 5$
d) $10 \times 10 \times 10$ e) 79×79 f) $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$



Hospital Administrator

The administrator of a busy hospital makes hundreds of decisions every day, many of which involve whole number computations and conversions. Should she purchase more 'standard' 24-tray carts for the orderlies to deliver meals to the new 84-bed hospital wing? The administrator calculates she would need four of these carts to take enough meals to the new wing. In the cart supplier's catalogue, there are also 32-tray carts which are more expensive, but priced within budget. Only three of the 32-tray carts would deliver all the meals, and they would cost less than four of the standard carts.



When the orderlies look at the catalogue picture of the 32-tray cart, they tell the administrator that the cart is too low. It is tiring and slower to bend so far down, so they wouldn't use the lowest eight tray bins. So, it's back to the calculator for the administrator. Which cart would *you* choose now? Give reasons for your answer.

Math Link

Your World

Carpet and tile prices are given per square unit. A paint can label tells the area the paint will cover in square units. Wallpaper is sold in rolls with area in square units.

Explore

Work on your own.

Blaise Pascal lived in France in the 17th century.

He was 13 years old when he constructed the triangle below.

This triangle is called Pascal's Triangle.



							row 1
							row 2
							row 3
							row 4
							row 5
							row 6

- What patterns do you see in the triangle?
- What symmetry do you see in the triangle?

Reflect & Share

Compare your patterns with those of a classmate.

Together, write about three different patterns you see in the triangle.

Connect

Here are some of the patterns in Pascal's Triangle.

		Sum
➤ Each row begins and ends with 1.	1	1
After the second row, each number	1 1	2
is the sum of the 2 numbers above it.	1 2 1	4
To write row 7:	1 3 3 1	8
Start with 1.	1 4 6 4 1	16
Add: $1 + 5 = 6$	$1 \downarrow 5 \downarrow 10 \downarrow 10 \downarrow 5 \downarrow 1$	32
Add: $5 + 10 = 15$	$1 \downarrow 6 \downarrow 15 \downarrow 20 \downarrow 15 \downarrow 6 \downarrow 1$	64
Add: $10 + 10 = 20$, and so on		

- The sum of the numbers in each row is shown above, and in the table on the next page.
From the 2nd row on, the sums can be written as powers.

Row	2	3	4	5	6	7
Sum in standard form	2	4	8	16	32	64
Sum in exponent form	2^1	2^2	2^3	2^4	2^5	2^6

We can use this table to predict the sum of the numbers in any row. All sums are powers of 2.

The exponent is 1 less than the row number.

So, the 10th row has sum: $2^9 = 512$

And the 19th row has sum: $2^{18} = 262\,144$

➤ The 3rd numbers in each row have this pattern: 1, 3, 6, 10, 15, ...

To get each term in the pattern, we add 1 more than we added before. We can use this to extend the pattern.

The 5th term: 15

The 6th term: $15 + 6 = 21$

The 7th term: $21 + 7 = 28$, and so on

Example

Describe each pattern in words. Write the next 3 terms.

a) 4, 9, 14, 19, ... b) 1, 3, 9, 27, ... c) 1, 3, 7, 13, 21, ...

Solution

a) 4, 9, 14, 19, ...

Start at 4.

Add 5 to get the next number.

The next 3 terms are 24, 29, 34.

b) 1, 3, 9, 27, ...

Start at 1.

Multiply by 3 to get the next number.

The next 3 terms are 81, 243, 729.

c) 1, 3, 7, 13, 21, ...

Start at 1.

Add 2.

Increase the number added by 2 each time.

The next 3 terms are 31, 43, 57.

Practice

- Write the next 3 terms in each pattern.

a) 7, 9, 11, 13, ...	b) 1, 5, 25, 125, ...
c) 4, 7, 10, 13, ...	d) 1, 10, 100, 1000, ...
e) 20, 19, 18, 17, ...	f) 79, 77, 75, 73, ...
- Write the next 3 terms in each pattern.

a) 3, 4, 6, 9, ...	b) 1, 4, 9, 16, ...
c) 101, 111, 121, 131, ...	d) 1, 12, 123, 1234, ...
e) 1, 4, 16, 64, ...	f) 256, 128, 64, 32, ...
- Describe each pattern in words.
Write the next 3 terms.

a) 200, 199, 201, 198, ...	b) 4, 7, 12, 19, ...
c) 100, 99, 97, 94, ...	d) 2, 6, 12, 20, ...
e) 50, 48, 44, 38, ...	f) 2, 6, 18, 54, ...
- Create your own number pattern. Trade patterns with a classmate. Describe your classmate's pattern.
Write the next 3 terms.
- a) Copy this pattern. Find each product.

$99 \times 11 = \square$	$99 \times 111 = \square$...
$99 \times 22 = \square$	$99 \times 222 = \square$...
$99 \times 33 = \square$	$99 \times 333 = \square$...
⋮	⋮	

- Extend this pattern sideways and down.
Predict the next 6 terms in each row and column.
 - Check your predictions with a calculator.
- This pattern shows the first 3 triangular numbers.



- Draw the next 3 terms in the pattern.
- List the first 6 triangular numbers.
- Find the next 2 triangular numbers without drawing pictures.
Explain how you did this.

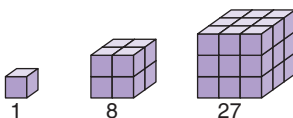
Number Strategies

One can of pop contains 355 mL. About how many 2-L pop bottles can be filled with one case of 24 cans of pop?

- d) Add consecutive triangular numbers;
that is, Term 1 + Term 2; Term 2 + Term 3; and so on.
What pattern do you see?
Write the next 3 terms in this pattern.
- e) Subtract consecutive triangular numbers;
that is, Term 2 – Term 1; Term 3 – Term 2; and so on.
What pattern do you see?
Write the next 3 terms in this pattern.



7. This pattern shows the first 3 cube numbers.



- a) Sketch the next 3 cube numbers in the pattern.
Use interlocking cubes if they help.
- b) Write the next 3 cube numbers without drawing pictures.
Explain how you did this.



8. Assessment Focus

- a) Write the first 10 powers of 2; that is, 2^1 to 2^{10} ,
in standard form.
- b) What pattern do you see in the units digits?
- c) How can you use this pattern to find the units digit of 2^{40} ?
- d) Investigate powers of other numbers.
Look for patterns in the units digits.
Explain how you can use these patterns to find units digits
for powers too large to display on the calculator.

Take It Further

9. Some sequences of numbers may represent different patterns.
Extend each pattern in as many different ways as you can.
Write the pattern rule for each pattern.
- a) 1, 2, 4, ... b) 1, 4, 9, ... c) 5, 25, ...

Reflect

Choose 3 different types of patterns from this section.
Describe each pattern.
Explain how you can use the pattern to predict the next term.

Using Different Strategies

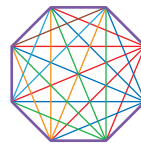
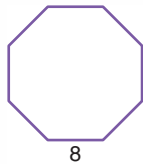
Problem

There are 8 people at a party.
Each person shakes hands with everyone else.
How many handshakes are there?

Think of a strategy

Strategy 1: **Draw a diagram and count.**

Draw 8 dots. Join every dot to every other dot.
Count the line segments.



$$5 + 5 + 4 + 3 + 2 + 1$$

$$\begin{aligned} \text{Number of line segments} \\ &= 8 + 5 + 5 + 4 + 3 + 2 + 1 \\ &= 28 \end{aligned}$$

There are 28 handshakes.

Strategy 2: **Solve simpler problems, then look for a pattern.**



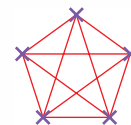
2 people
1 handshake



3 people
3 handshakes



4 people
6 handshakes



5 people
10 handshakes

Make a table.

Each time you add a person,
you add one more

handshake than the time before.

So, 6 people: $10 + 5$, or 15 handshakes

7 people: $15 + 6$, or 21 handshakes

8 people: $21 + 7$, or 28 handshakes

Number of People	Number of Handshakes
2	1
3	3
4	6
5	10

+2
+3
+4





Strategy 3: Use reasoning.

Each of 8 people shakes hands with 7 other people.
That is, 8×7 , or 56 handshakes
But we have counted each handshake twice.
We have said that A shaking hands with B is different from B shaking hands with A.
So, we divide by 2: $\frac{56}{2} = 28$
There are 28 handshakes.

Look back

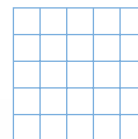
- Look at the numbers of handshakes for 2 to 8 people.
Where have you seen this pattern before?
- What if there were 9 people at the party?
How many handshakes would there be?
How do you know?

Problems

Solve each problem. Try to use more than one strategy.



1. A ball is dropped from a height of 16 m.
Each time it hits the ground, the ball bounces to one-half its previous height. The ball is caught when its greatest height for that bounce is 1 m. How far has the ball travelled?
2. A rectangular garden is 100 m long and 44 m wide.
A fence encloses the garden.
The fence posts are 2 m apart.
How many posts are needed?
3. Here is a 5 by 5 square.
How many squares of each different size can you find in this large square?

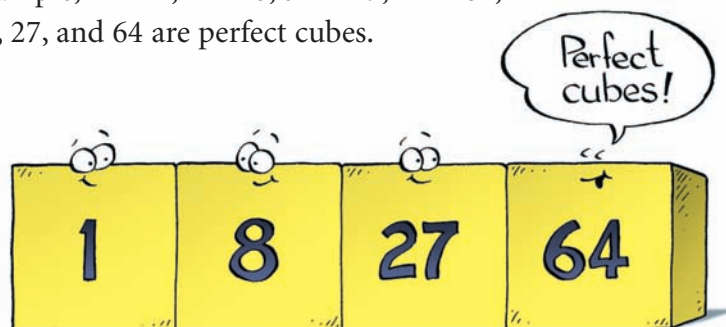


Reflect

Why might you want to solve a problem more than one way?

What Do I Need to Know?

- ✓ A *factor* of a number divides into the number exactly; that is, there is no remainder.
For example, $6 \div 2 = 3$, so 2 is a factor of 6.
- ✓ A *prime number* has only 2 factors, itself and 1.
For example, the only factors of 17 are 17 and 1, so 17 is a prime number.
- ✓ A *composite number* has more than 2 factors.
For example, 12 has factors 1, 2, 3, 4, 6, and 12, so 12 is a composite number.
- ✓ A *square number*, or *perfect square*, has an odd number of factors.
It can also be written as a power with exponent 2.
For example, the factors of 9 are 1, 3, and 9, so 9 is a perfect square.
We write $9 = 3^2$.
- ✓ A *square root* of a number is a factor that is squared to get the number.
For example, 9 is a square root of 81 because $9^2 = 81$.
We write $\sqrt{81} = 9$.
- ✓ When a number is written in *exponent form*, it is written as a *power*.
For example, for the power 5^3 :
5 is the *base*.
3 is the *exponent*.
 $5 \times 5 \times 5$ is the *expanded form*.
125 is the *standard form*.
- ✓ A *cube number*, or *perfect cube*, is a power with exponent 3.
For example, $1^3 = 1$, $2^3 = 8$, $3^3 = 27$, $4^3 = 64$,
so 1, 8, 27, and 64 are perfect cubes.



What Should I Be Able to Do?

For extra practice, go to page 438.

LESSON

- 1.1** 1. Find each answer.

Use pencil and paper.

- a) $3621 + 8921$
- b) $5123 - 4123$
- c) 35×12
- d) 125×27
- e) $815 + 642 - 85$
- f) $1638 \div 21$

2. This table shows the highest all-time scorers at the end of the 2000–2001 NBA season.

Kareem Abdul-Jabbar	38 387
Karl Malone	32 919
Wilt Chamberlain	31 419
Michael Jordan	29 277

- a) What is the total number of points?
 - b) Write a problem about these data. Solve your problem. Justify the strategy you used.
3. a) Write the number 300 as the sum of 2 or more consecutive whole numbers. Find as many ways to do this as you can.
- b) What patterns do you see in the numbers added?
- c) Suppose you started with another 3-digit number. Will you see similar patterns? Investigate to find out.

4. Solve each problem. State any assumption you made.
- a) Armin's house is 3 km from a mall. He walks 1 km in 15 min. How long does it take Armin to walk to the mall?
 - b) Tana makes \$15, \$21, and \$19 for baby-sitting one weekend. How much will Tana make in a month?
5. The table shows the ticket prices and number of tickets sold for a popular movie.

	Ticket Price (\$)	Number of Tickets Sold
Adults	12	125
Seniors	10	34
Youths	8	61

Calculate the total cost of the ticket sales.

- 1.2** 6. Find all the factors of each number.
- a) 36 b) 50
 - c) 75 d) 77
7. Find the first 10 multiples of each number.
- a) 9 b) 7
 - c) 12 d) 15
8. For the numbers 18 and 60, find:
- a) the GCF b) the LCM
- Draw a Venn diagram to illustrate part a.

- 9.** How many prime numbers are even?
Justify your answer.

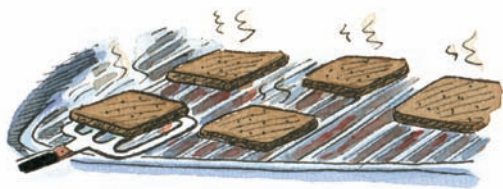
- 1.3 10.** Find a square root of each number.
a) 121 b) 169 c) 225

- 11.** Find each square root.
Draw a picture if it helps.
a) $\sqrt{25}$ b) $\sqrt{100}$ c) $\sqrt{81}$

- 12.** Calculate the area of a square with each side length.
a) 7 cm b) 17 cm c) 93 m

- 13.** The area of a square is 81 m^2 .
What is the perimeter of the square? How do you know?

- 14.** Raquel cooks 8-cm square hamburgers on a grill.
The grill is a rectangle with dimensions 40 cm by 40 cm.
How many hamburgers can be grilled at one time?
Justify your answer.



- 1.4 15.** Copy and complete this table.

	Exponent Form	Base	Exponent	Expanded Form	Standard Form
a)	3^4				
b)	2^5				
c)	10^7				
d)		5	4		
e)				$4 \times 4 \times 4 \times 4$	

- 1.5 16.** A perfume formula requires 4 g of an essential oil per bottle.
a) How many grams are needed for 2500 bottles?
b) Write this number in exponent form.



- 17.** Write these numbers in order from greatest to least.
 $3^4, 4^4, 5^3, 2^6$

- 18. a)** Write the next 3 terms in each pattern.
i) 3, 5, 6, 8, 9, ...
ii) 1, 2, 4, 8, ...
iii) 1, 4, 9, 16, ...
iv) 3, 4, 6, 9, ...
b) Describe each pattern in part a.



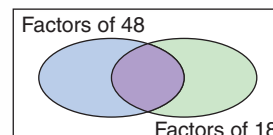
- 19. a)** Copy and complete this pattern.
 $1^2 + 2^2 = \square$
 $2^2 + 3^2 = \square$
 $3^2 + 4^2 = \square$
 $4^2 + 5^2 = \square$
b) Write the next two rows in the pattern.
c) Describe the pattern.

- 20.** $1^2 = 1$
 $1^2 + 2^2 = 5$
 $1^2 + 2^2 + 3^2 = 14$
 $1^2 + 2^2 + 3^2 + 4^2 = 30$
a) Write the next two lines in the pattern.
b) What pattern do you see?

Practice Test

1. Estimate. Describe your strategy.
a) $624 + 1353$ b) $897 \div 23$ c) 752×36
2. Use mental math to evaluate $2 \times 395 \times 5$. Explain your strategy.

3. Use a Venn diagram to show the factors of 48 and 18. Circle the GCF.



4. Use patterns to find the first 6 common multiples of 15 and 6.
5. Use the clues below to find the mystery number. Explain your strategy and reasoning.
Clue 1: I am a 2-digit number.
Clue 2: I am less than 9^2 .
Clue 3: I have 26 and 6 as factors.
6. Sharma plays basketball every third day of the month. She baby-sits her little brother every seventh day of the month. How many times in a month will Sharma have a conflict between basketball and baby-sitting? Explain your thinking.
7. Write these numbers in order from least to greatest.
a) $5^2, 2^5, \sqrt{25}, 10^2, 3^3$ b) $10 \times 10 \times 10, 2^3, \sqrt{400}, 3^2, 17$
8. The perimeter of a square is 32 cm. What is the area of the square? Explain your thinking. Include a diagram.
9. Write the next 3 terms in each pattern. Describe each pattern.
a) 1, 3, 6, 10, ... b) 23, 25, 27, ... c) 100, 81, 64, 49, ...
10. Write the number 35 as:
a) the sum of 3 squares
b) the difference between 2 squares
c) the sum of a prime number and a square

One of the greatest mathematicians of the Middle Ages was an Italian, Leonardo Fibonacci.

Fibonacci is remembered for this problem:

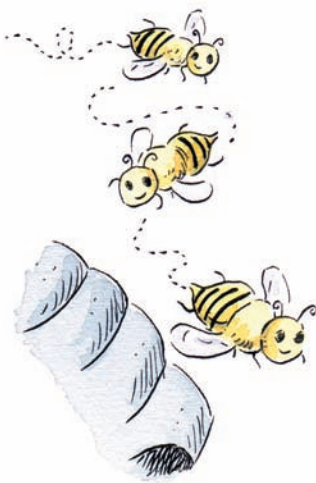
A pair of rabbits is placed in a large pen.
 When the rabbits are two months old, they produce another pair of rabbits.
 Every month after that, they produce another pair of rabbits.
 Each new pair of rabbits does the same.
 None of the rabbits dies.
 How many rabbits are there at the beginning of each month?

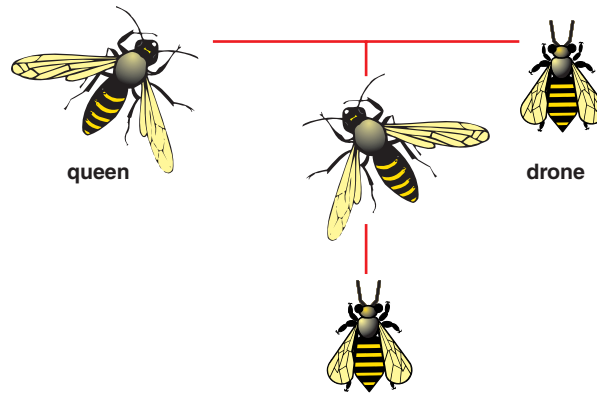


This table shows the rabbits at the beginning of the first 5 months.

Beginning of Month	Number of Pairs	Number of Rabbits
1	1	
2	1	
3	2	
4	3	
5	5	

- Continue the pattern for two more months. Use different colours to show the new rabbits.
 - Write the number of pairs of rabbits at the beginning of each month, for the first 7 months. These are the Fibonacci numbers. What pattern do you see? Explain how to find the next number in the pattern.
- The Fibonacci sequence appears in the family tree of the drone, or male bee. The drone has a mother, but no father. Female bees are worker bees or queens. They have a mother and a father. The family tree of a male bee back to its grandparents is shown on the next page.





Check List

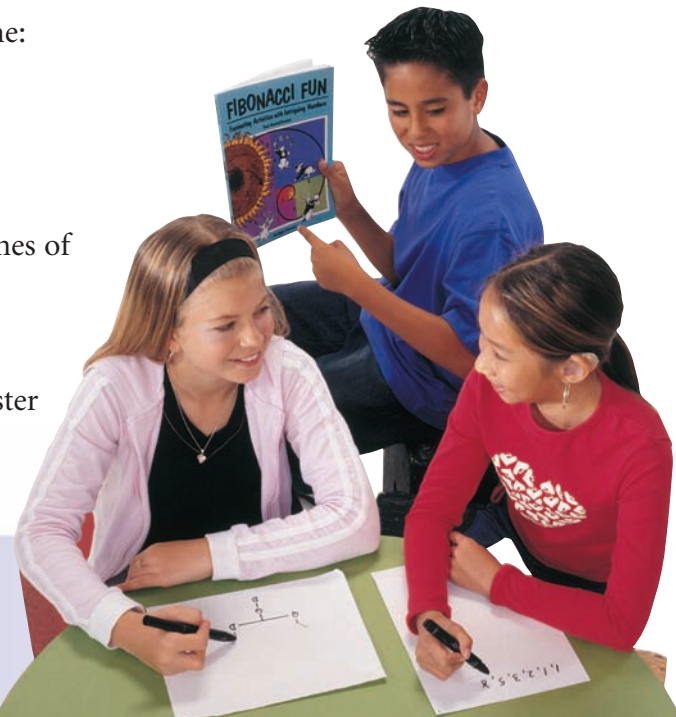
Your work should show:

- ✓ how you used patterns to find your answers
- ✓ all diagrams and charts, clearly presented
- ✓ a clear explanation of your results
- ✓ your understanding of Fibonacci numbers

- a) Copy the diagram. Trace the male bee's ancestors back 5 more generations.
- b) Explain how you can use this pattern to find the Fibonacci numbers.

There are many patterns you can find in the Fibonacci sequence.

3. Write the first 15 Fibonacci numbers.
 - a) What type of number is every third number?
 - b) Which number is a factor of every fourth number?
 - c) Which number is a factor of every fifth number?
4. Add the squares of the:
 - 2nd and 3rd terms
 - 3rd and 4th terms
 - 4th and 5th terms
 What do you notice?
 Write the next two lines of this pattern.
5. Research Fibonacci numbers. Make a poster to show your work.



Reflect on the Unit

What have you learned about whole numbers?
 What have you learned about number patterns?
 Write about some of the things you have learned.