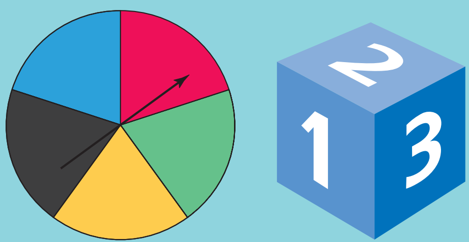
UNIT 11 Probability

Many games involve probability and chance. One game uses this spinner or a number cube labelled 1 to 6.



You can choose to spin the pointer or roll the number cube. You win if the pointer lands on red. You win if you roll a 6. Are you more likely to win if you spin the pointer or roll the number cube? Explain.

What You'll Learn

- Use the language of probability.
- Conduct simple experiments.
- List the possible outcomes of experiments by using tree diagrams, modelling, and lists.
- Identify possible outcomes and favourable outcomes.
- State the probability of an outcome.
- Understand how probability can relate to sports and games of chance.
- Use probability to solve problems.

Why It's Important

In the media, you hear and read statements about the probability of everyday events, such as living to be 100 or winning the lottery.
To make sense of these statements, you need to understand probability.

Key Words

- probability
- outcome
- tree diagram
- relative frequency
- experimental probability
- theoretical probability

UNIT 11

Skills You'll Need

Converting Fractions and Decimals to Percents

Percent (%) means "per hundred" or out of one hundred.

Example

Express each fraction as a decimal, then as a percent.

a) $\frac{9}{50}$ b) $\frac{1}{4}$ c) $\frac{5}{8}$ d) $\frac{7}{16}$

Solution

To convert a fraction to a decimal, try to write an equivalent fraction with denominator 100.

a) $\frac{9}{50} = \frac{18}{100}$ (× 2 numerator and denominator)

b) $\frac{1}{4} = \frac{25}{100}$ (× 25 numerator and denominator)

$\frac{18}{100} = 0.18$, or 18%

$\frac{25}{100} = 0.25$, or 25%

When you cannot write an equivalent fraction, use a calculator to divide.

c) $\frac{5}{8} = 5 \div 8$
$= 0.625$
$= 62.5\%$

d) $\frac{7}{16} = 7 \div 16$
$= 0.4375$
$= 43.75\%$

Check

1. Express each decimal as a percent.

a) 0.1 b) 0.01 c) 0.24 d) 0.05

2. Express each fraction as a decimal, then as a percent.

a) $\frac{7}{10}$ b) $\frac{3}{5}$ c) $\frac{9}{25}$ d) $\frac{3}{4}$

3. Express each fraction as a decimal, then as a percent.

a) $\frac{7}{40}$ b) $\frac{3}{8}$ c) $\frac{13}{16}$ d) $\frac{51}{200}$

11.1 Listing Outcomes

Focus Investigate outcomes of experiments.

When you roll a number cube, the outcomes are equally likely.
For a spinner with sectors of equal areas, when the pointer is spun, the outcomes are equally likely.

Explore

Work with a partner. You will need a number cube labelled 1 to 6, and a spinner similar to the one shown below.

List the possible outcomes of rolling the number cube and spinning the pointer.
How many outcomes include rolling a 4?
How many outcomes include landing on red?
How many outcomes have an even number on the cube and the pointer landing on blue?

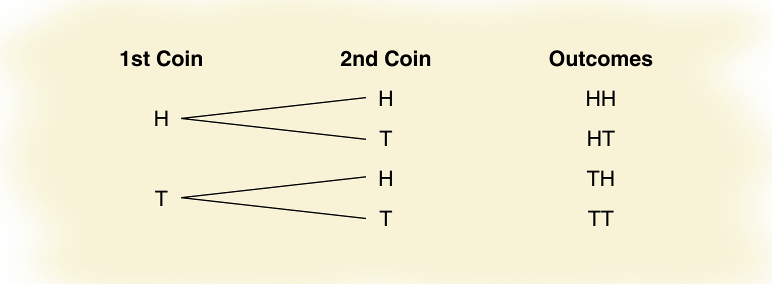
Reflect & Share

Compare the strategy you used to find the outcomes with that of another pair of classmates.
Was one strategy more efficient than another? Explain.

Connect

An outcome is the possible result of an experiment or an action.
When a coin is tossed, the possible outcomes are heads or tails.
To show the possible outcomes for an experiment that has two or more actions, we can use a **tree diagram**.

When 2 coins are tossed, the outcomes for each coin are heads (H) or tails (T).
List the outcomes of the first coin toss.
For each outcome, list the outcomes of the second coin toss.
Then list the outcomes for the coins tossed together.



There are 4 possible outcomes: HH, HT, TH, TT

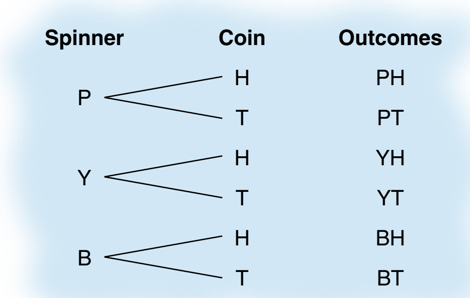
Example

Farah tosses a coin and spins the pointer on this spinner.

a) Draw a tree diagram to show the possible outcomes.
b) List all the possible outcomes.
c) How many outcomes include the pointer landing on pink?
d) How many outcomes include tails?

Solution

a) The sectors on the spinner have equal areas, so the outcomes are equally likely. The possible outcomes for the spinner are: pink (P), yellow (Y), or blue (B). For each colour, the possible outcomes for tossing the coin are: heads (H) or tails (T)



b) The outcomes are: pink/heads, pink/tails, yellow/heads, yellow/tails, blue/heads, blue/tails
c) There are two outcomes with the colour pink: pink/heads, pink/tails
d) There are three outcomes with tails: pink/tails, yellow/tails, blue/tails

Practice

1. List the possible outcomes in each case.
   a) spinning the pointer

   b) rolling a number cube labelled 1 to 6

2. List the possible outcomes in each case.
   a) the colour of a traffic light when you reach it
   b) the gender of a baby who is born
   c) the points scored in a hockey game
   d) the suit of a card pulled from a deck of playing cards

3. The Scenic Railroad sells tickets for trips on Saturdays and Sundays. All-day and half-day trips are available. There are adult, child, and senior fares. Draw a tree diagram to show the possible ticket types.

4. Use a tree diagram to show the possible combinations for breakfast. You must choose one of each:
   - eggs or fruit
   - toast, pancakes, or cereal
   - milk or juice

5. Jim has to choose an outfit. His choices of pants are black, grey, or navy. His sweater choices are red, beige, white, or yellow.
   a) Draw a tree diagram to display all the possible outfits.
   b) How many outfits have either black pants or a white sweater?
   c) How many outfits do not have black pants or do not have a white sweater?
   d) How many outfits have a black sweater?

6. A deli offers 2 soups, 4 salads, 5 sandwiches, and 3 beverages. How many choices are there for a customer who wants each of the following meals?
   a) a salad and a beverage
   b) soup, a sandwich, and a beverage
   c) soup, a salad, and a beverage
   d) a sandwich or salad, and a soup

Mental Math

Simplify.
- $(-3) + (+5)$
- $(-3) + (-5)$
- $(+3) + (-5)$
- $(+3) - (+5)$

7. **Assessment Focus**

a) Copy and complete this table.
Show the sums when two number cubes are rolled.

When one cube shows 1 and the other cube shows 4, then the sum is 5.

Sum of Numbers on Two Cubes						
Number on Cube	1	2	3	4	5	6
1	2	3	4	5	6	7
2						
3						
4						
5						
6						

b) How many different outcomes are there for the sum of the numbers on the cubes?
c) In how many ways can the sum be 6?
d) In how many ways can the sum be 9?
e) In how many ways can the sum be 2 or 12?
f) Why do you think 7 is a lucky number?
g) Draw a tree diagram to show these results. Why do you think a table was used instead of a tree diagram?

Show your work.

Take It Further

8. A lock combination comprises the four digits from 1 to 4 in any order.
How many possible combinations are there in each case?
a) The digits cannot repeat within the code.
b) The digits can repeat within the code.

Reflect

Explain why a tree diagram is helpful to list the outcomes of an experiment.

11.2 Experimental Probability

Focus Determine probability from experiments.

Explore

Work with a partner.
You will need a coin.
When you toss a coin, which outcome do you think is more likely?
Do you think the outcomes are equally likely? Explain.

Outcome	Tally	Frequency
Heads		
Tails		

➢ Toss the coin 50 times.
How many times do you think you will get heads?
Record the results in a table.

➢ Write the number of heads as a fraction of the total number of tosses.
Write the number of tails as a fraction of the total number of tosses.
Add the fractions. What do you notice?

Reflect & Share

How do the results compare with your prediction?
Combine your results with those of another pair of classmates.
This is same as tossing the coin 100 times.
Write the new fractions for 100 tosses.
Add the fractions. What do you notice?

Connect

The **relative frequency** is the number of times an outcome occurs divided by the total number of times the experiment is conducted.

$$\text{Relative frequency} = \frac{\text{Number of times an outcome occurs}}{\text{Number of times experiment is conducted}}$$

The relative frequency may be written as a fraction, a decimal, or a percent. Relative frequency is also called **experimental probability**.

When a thumbtack is dropped, it can land with its point up or on its side.
Here are the results of 100 drops:

Outcome	Frequency
Point up	46
On its side	54

Relative frequency of Point up $= \dfrac{\text{Number of times Point up}}{\text{Total number of drops}}$

$= \dfrac{46}{100}$, or 0.46

Relative frequency of On its side $= \dfrac{\text{Number of times On its side}}{\text{Total number of drops}}$

$= \dfrac{54}{100}$, or 0.54

Outcome	Frequency	Relative Frequency
Point up	46	0.46
On its side	54	0.54

The sum of the relative frequencies for an experiment is 1.
That is, $\dfrac{46}{100} + \dfrac{54}{100} = 0.46 + 0.54$
$= 1$

Example

The number of times a player goes up to bat is referred to as the player's "at bats."
This table shows the number of at bats and hits for some of the greatest players in the Baseball Hall of Fame.

In baseball, a "batting average" is a relative frequency.

a) Calculate the batting average for each player.
b) Order the players from greatest to least batting average.

Players in Baseball Hall of Fame		
Player	**At Bats**	**Hits**
Aaron	12 364	3771
Cobb	11 429	4191
Gehrig	8 001	2721
Jackson	9 864	2584
Mantle	8 102	2415
Mays	10 881	3283

Solution

a) To calculate each player's batting average, divide the number of hits by the number of at bats.
Round each batting average to 3 decimal places.

Use a calculator to write each fraction as a decimal.

Aaron $= \dfrac{3771}{12\,364} \doteq 0.305$

Cobb $= \dfrac{4191}{11\,429} \doteq 0.367$

Gehrig $= \dfrac{2721}{8001} \doteq 0.340$

Jackson $= \dfrac{2584}{9864} \doteq 0.262$

Mantle $= \dfrac{2415}{8102} \doteq 0.298$

Mays $= \dfrac{3283}{10\,881} \doteq 0.302$

b) The batting averages, from greatest to least, are:
0.367, 0.340, 0.305, 0.302, 0.298, 0.262
The players, from greatest to least batting average, are:
Cobb, Gehrig, Aaron, Mays, Mantle, and Jackson

Practice

Name	At Bats	Hits
Yang Hsi	58	26
Aki	41	20
David	54	23
Yuk Yee	36	11
Eli	49	18
Aponi	42	15
Leah	46	22
Devadas	45	17

1. This table shows data for a baseball team.
Find the batting average of each player.
Round each answer to 3 decimal places.

2. Write each relative frequency as a decimal to 3 decimal places.
   a) A telemarketer made 200 phone calls and 35 new customers signed up. What is the relative frequency of getting a customer? Not getting a customer?
   b) A quality controller tested 175 light bulbs and found 5 defective. What is the relative frequency of finding a defective bulb? Finding a good bulb?

3. A paper cup is tossed.
The cup lands with the top up 27 times, the top down 32 times, and on its side 41 times.
   a) What are the possible outcomes of tossing a paper cup?
   b) Are the outcomes equally likely? Explain.
   c) State the relative frequency of each outcome.

4. a) Conduct the paper cup experiment in question 3.
Decide how to hold the cup to drop it.
Repeat the experiment until you have 100 results.
   b) Compare your results with those from question 3.
Are the numbers different? Explain.

5. Use 3 red counters and 3 yellow counters.
You may place some or all of the counters in a bag.
You then pick a counter without looking.
How many different ways can you place the counters in the bag so you are more likely to pick a red counter than a yellow counter? Explain.

6. Copy and continue the table to show all months of the year. Have each student write her or his month of birth on the board. Find the number of students who were born in each month.

a) Complete the table.

Month	Tally	Frequency	Relative Frequency
January			
February			
March			

b) What is the relative frequency for birthdays in the same month as yours?

c) Find the sum of the relative frequencies. Explain why this sum makes sense.

Number Strategies

Find each percent.
- 10% of $325.00
- 15% of $114.00
- 20% of $99.99
- 25% of $500.00

7. **Assessment Focus** A regular octahedron has faces labelled 1 to 8. Two of these octahedra are rolled. The numbers on the faces the octahedra land on are added. Work with a partner. Use the regular octahedra you made in Unit 3. Label the faces of each octahedron from 1 to 8.

Conduct an experiment to find the relative frequency of getting a sum of 7 when two regular octahedra are rolled.

a) Report your results.

b) How are the results affected if you conduct the experiment 10 times? 50 times? 100 times? Explain.

Reflect

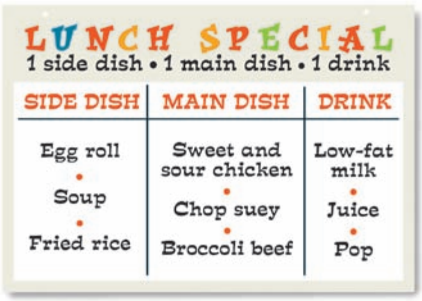
Suppose you know the relative frequency of one outcome of an experiment.
How can you use that to predict the likelihood of that outcome occurring if you conduct the experiment again?
Use an example to explain.

Mid-Unit Review

LESSON

11.1 **1.** Jenna plays a video game on her computer. Each time she plays, she can choose an easy, intermediate, or challenging level of difficulty. She can also choose 1 or 2 players. Use a tree diagram to show the possible game choices.

2. Use a tree diagram to show the possible lunch choices.



11.2 **3.** Write each relative frequency as a decimal.

a) An air traffic controller's records show 512 planes landed one day. Seventeen planes were 727s. What is the relative frequency of a 727 landing?

b) A cashier served 58 customers in one shift. Thirty-two customers paid cash. What is the relative frequency of a customer paying cash?

c) Qam spun a pointer on a spinner 95 times. The pointer landed on purple 63 times. What is the relative frequency of landing on purple?

4. You will need 4 cubes: 2 of one colour, 2 of another colour; and a bag. Place the cubes in the bag. Pick 2 cubes without looking. Design and conduct an experiment to find the relative frequency of choosing 2 matching cubes.

5. A number cube is labelled 1 to 6.

a) What are the possible outcomes when this cube is rolled?

b) Are these outcomes equally likely? Explain.

c) Design and conduct an experiment to find the relative frequency of each outcome.

d) Do the results confirm your prediction in part b? Explain.

e) How does your answer to part d depend on the number of times you roll the number cube? Explain.

6. There are 3 blue counters and 3 green counters in a bag. You may add to the bag or remove from the bag, as listed below. You put:

a) 1 red counter in the bag

b) 1 more green counter in the bag

c) 2 blue counters in the bag

You then pick a counter without looking. Which of the actions above would make it more likely that you would pick a green counter? Explain.

11.3 Theoretical Probability

Focus Explore and apply theoretical probability.

Explore

Work in a group.
A carnival game has a bucket of different-coloured balls.
Each player is asked to predict the colour of the ball he or she will select.
The player then selects a ball, without looking.
If the guess is correct, the player wins a prize.
After each draw, the ball is returned to the bucket.

Use linking cubes.
Put 4 red, 3 blue, 2 yellow, and 1 green cube in a bag.
Suppose you take 1 cube without looking.
Predict the probability that you will pick each colour.
Play the game 50 times.
What is the experimental probability for picking each colour?
How does each predicted probability compare with the experimental probability?

Reflect & Share

Combine your results with those of another group of students.
How does each experimental probability compare with the predicted probability now?

Connect

Recall that when the outcomes of an experiment are equally likely, the probability of any outcome is:

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$$

This is called the **theoretical probability**, but we usually say "probability."
You find the probability by analysing the possible outcomes rather than by experimenting.

When you pick an object without looking, the object is picked **at random.**

Twenty counters were put in a bag:
7 green, 6 black, 5 orange, and 2 purple
You take one counter from the bag without looking.
There are 4 outcomes: green, black, orange, and purple
Suppose the favourable outcome is black.
Then, the probability of picking a black counter is: $\frac{6}{20} = 0.3$

Suppose the favourable outcome is green.
Then, the probability of picking a green counter is: $\frac{7}{20} = 0.35$

Note that the outcomes are not equally likely.

We can predict the possible number of times an outcome will occur by multiplying the probability by the number of repetitions.
Suppose we pick a counter at random 54 times.
Then, the predicted number of times a black counter is picked is:
$54 \times 0.3 = 16.2$
We would expect to pick a black counter about 16 times.
However, we may never pick a black counter, or we might always pick a black counter.

Example

Suppose you roll a number cube 100 times. Predict how many times:

a) a 1 will show **b)** a 5 will show
c) a 1 or a 5 will show **d)** a 1 or a 5 will not show

How are the answers to parts c and d related?

Solution

When a number cube is rolled, there are six possible outcomes.
The outcomes are equally likely.

a) The probability of rolling a 1 is $\frac{1}{6}$.
So, the predicted number of times a 1 will show in 100 rolls is:
$\frac{1}{6} \times 100 = \frac{100}{6} \doteq 17$

b) The probability of rolling a 5 is also $\frac{1}{6}$.
So, the predicted number of times a 5 will show is also about 17.

c) The probability of rolling a 1 or a 5 is $\frac{2}{6}$, or $\frac{1}{3}$.
So, the predicted number of times a 1 or a 5 will show is:
$\frac{1}{3} \times 100 = \frac{100}{3} \doteq 33$

d) For a 1 and a 5 not to show, a 2, 3, 4, or 6 shows.
The probability of rolling a 2, 3, 4, or 6 is $\frac{4}{6}$, or $\frac{2}{3}$.
So, the predicted number of times a 1 or a 5 does not show is:
$\frac{2}{3} \times 100 = \frac{200}{3} \doteq 67$

The predicted number of times a 1 or a 5 shows and the predicted number of times a 1 or a 5 does not show are:
$33 + 67 = 100$
An outcome occurs or it does not occur. So,

Predicted number of times an outcome occurs	+	Predicted number of times the outcome does not occur	=	Number of times the experiment is conducted

In the *Example* part c, in 100 rolls, a 1 or a 5 will show about 33 times. This does not mean that a 1 or a 5 will show *exactly* 33 times, but the number of times will likely be close to 33.

The more times an experiment is conducted, the closer the experimental probability is to the theoretical probability.

Practice

1. A bag contains these granola bars: 12 apple, 14 banana, 18 raisin, and 10 regular. You pick one bar at random. Find the probability of choosing:

a) a banana granola bar **b)** an apple granola bar

When you see the word "probability" in a sentence, it means theoretical probability.

2. There are 8 names in a hat.
You pick one name without looking. Find each probability.

a) A three-letter name will be picked.
b) A five-letter name will be picked.
c) Laura will be picked.
d) Jorge will not be picked.

Number Strategies

Simplify.

- $\frac{5}{8} + \frac{3}{4}$
- $\frac{4}{5} - \frac{2}{3}$
- $1\frac{3}{10} + 2\frac{1}{2}$
- $\frac{2}{3} + \frac{2}{5} + \frac{3}{10}$

3. Is each statement true or false? Explain.

a) If you toss a coin 10 times, you will never get 10 heads.

b) If you toss a coin 10 times, you will always get exactly 5 heads.

c) If you toss a coin many times, the number of heads should be approximately $\frac{1}{2}$ the number of tosses.

4. The pointer on this spinner is spun 100 times.
Is each statement true or false? Justify your answer.

a) The pointer will land on *Win* about 33 times.

b) The pointer will land on *Win*, *Lose*, and *Tie* an equal number of times.

c) The pointer will land on *Lose* exactly 33 times.

5. Two hundred fifty tickets for a draw were sold.
The first ticket drawn wins the prize.

a) Joe purchased 1 ticket. What is the probability Joe will win?

b) Maria purchased 10 tickets.
What is the probability Maria will win?

c) Ivan purchased 25 tickets.
What is the probability Ivan will *not* win?

6. Assessment Focus

a) Construct a spinner with red, yellow, blue, and green sectors, so the following probabilities are true.
- The probability of landing on red is $\frac{1}{4}$.
- The probability of landing on yellow is $\frac{1}{2}$.
- The probability of landing on blue is $\frac{1}{6}$.
- The probability of landing on green is $\frac{1}{12}$.

Explain how you drew your spinner.

b) In 200 trials, about how many times would the pointer land on each colour?

c) Suppose the spinner had been constructed so the probability of landing on yellow was $\frac{1}{4}$.
What effect would this have on the probability of landing on each other colour? Explain.

Reflect

How are theoretical probability and experimental probability similar? Different?
Use an example to explain.

11.4 Applications of Probability

Focus Solve probability problems.

Explore

Work with a partner.
You will play the *Sum and Product* game.
You will need 4 blank cards and a bag.
Write the numbers from 1 to 4 on the cards.
Place the cards in the bag.
Each person picks a card.
Both of you find the sum and the product of the two numbers.
One of you is Player A, the other is Player B.
If the sum is less than or equal to the product, Player A gets a point.
If the sum is greater than the product, Player B gets a point.

- ➢ Who is likely to win? Explain your reasoning.
- ➢ Play the game several times; you choose how many times. Who won?
- ➢ How does your prediction of the winner compare with your result?

Reflect & Share

Compare your results with those of another pair of classmates.
Work together to come up with an explanation of who is more likely to win.

Connect

- ➢ Probability can be expressed as a fraction, a decimal, or a percent. When probability is expressed as a percent, we use the word "chance."
  For example, the weather forecast is a 40% chance of rain today.
  This means that the probability of rain is: $\frac{40}{100} = 0.4$
- ➢ When an outcome is certain, the probability of it occurring is 1. For example, when we toss a coin, the probability of it landing heads or tails is 1.
  When an outcome is impossible, the probability of it occurring is 0. For example, when we roll a number cube labelled 1 to 6, the probability of a 7 showing is 0.

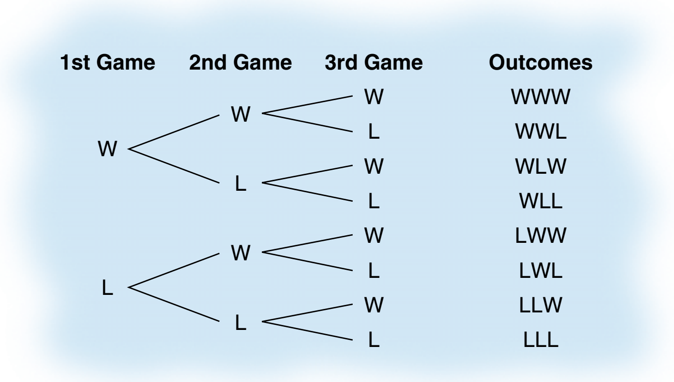
Example

Flick This is an Ultimate Frisbee team.
The team plays 3 games against 3 other teams.
All 4 teams have equal chances of winning.

a) What is the chance that Flick This will win all three of its games?
b) What is the chance that Flick This will win exactly one game?
c) What is the chance that Flick This will win at least two games?

Solution

For any game, the possible outcomes are win (W) or lose (L). These outcomes are equally likely. Draw a tree diagram to list the possible results of 3 games for Flick This.



There are 8 possible outcomes.

a) There is 1 outcome in which Flick This wins all three games: WWW
So, the probability of 3 wins is: $\frac{1}{8} = 0.125$
So, the chance of winning 3 games is 12.5%.

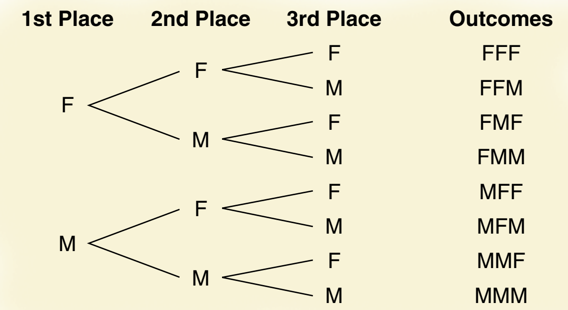
The word "exactly" is included because "winning one game" might be interpreted as winning one or more games.

b) There are 3 outcomes in which Flick This wins exactly one game: WLL, LWL, LLW
So, the probability of winning exactly 1 game is: $\frac{3}{8} = 0.375$
So, the chance of winning exactly 1 game is 37.5%.

c) There are 4 outcomes in which Flick This wins at least two games: WWW, WWL, WLW, LWW
So, the probability of winning at least 2 games is: $\frac{4}{8} = \frac{1}{2} = 0.5$
So, the chance of winning at least 2 games is 50%.

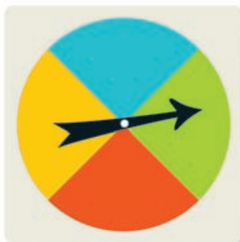
Practice

1. The 1st, 2nd, and 3rd place winners of a contest can be female or male. This tree diagram shows the possible outcomes of the contest.



a) How many possible outcomes are there?
b) What is the probability that all the winners are female?
c) What is the probability that none of the winners is male?
d) How are the answers to parts b and c related? Explain.

2. On this spinner, the pointer is spun once.
The colour is recorded.
The pointer is spun a second time.
The colour is recorded.



a) Suppose you win if you spin the same colour on both spins.
What are your chances of winning?
b) Suppose you win if you spin two different colours.
What are your chances of winning?

3. **a)** Three coins are tossed. Find the chance of tossing:
i) one heads and two tails **ii)** exactly two heads
iii) at least two tails **iv)** no heads
b) Why do we need the words "at least" in part a, iii?
What if these words were left out?
How would the answer change?
c) Why do we need the word "exactly" in part a, ii? What if this word was left out? How would the answer change?

4. At a carnival, the game with the least chance of winning often has the greatest prize. Explain why this might be.

5. There are four children in a family.
What is the chance of each event?
a) There are two boys and two girls.
b) There is at least one girl.
c) All four children are of the same gender.

6. **Assessment Focus** The school cafeteria has this lunch menu.
A student chooses a sandwich and a vegetable. Assume the choice is random.
a) Find the probability of each possible combination.
b) Suppose 3 desserts were added to the menu. Each student chooses a sandwich, a vegetable, and a dessert.
How would the probabilities of possible combinations change?
Use examples to explain your thinking.

Lunch Menu

Sandwich	Vegetable
Grilled Cheese	Broccoli
Chicken	Carrots
Tuna	

Take It Further

7. At the school carnival, there is a game with two spinners.

Spinner A

Spinner B

You get two spins.
You may spin the pointer on each spinner once, or spin the pointer on one spinner twice.
If you get pink on one spin and yellow on another spin (the order does not matter), you win.
To have the greatest chance of winning, what should you do? Explain.

Calculator Skills

Which three consecutive prime numbers have a product of 7429 and a sum of 59?

Reflect

How is probability related to chance?
Use an example in your explanation.

Choosing a Strategy

Strategies

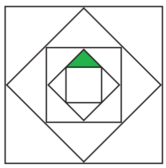
- Make a table.
- Use a model.
- Draw a diagram.
- Solve a simpler problem.
- Work backward.
- Guess and check.
- Make an organized list.
- Use a pattern.
- Draw a graph.
- Use logical reasoning.

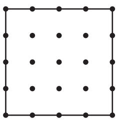
1. For her birthday, Janine was given a row of 25 pennies.
   Her father replaced every second coin with a nickel.
   Her mother replaced every third coin with a dime.
   Her brother replaced every fourth coin with a quarter.
   Her uncle replaced every fifth coin with a loonie.
   How much did Janine get on her birthday?

2. Arif has a part-time job.
   He was offered $96 per week or $4.50/h.
   Which is the better deal? Explain.

3. a) The perimeter of a rectangle is 36 cm.
   What is the maximum possible area of the rectangle?
   b) The sum of the length, width, and height of a rectangular prism is 18 cm.
   What is the maximum possible volume of the prism?

4. What fraction of this figure is shaded?





5. Divide the square at the left into four congruent figures.
   Record each way you find on dot paper.
   Find at least ten different ways to do this.

6. Running shoes cost $79.99. They are on sale for 20% off.
   The sales tax of 15% has to be added.
   Which would you choose? Explain.
   a) Take the 20% off the price, then add the 15% sales tax.
   b) Add the 15% sales tax, then take off the 20%.

7. The Magic Money Box doubles any amount of money placed in it, then adds $1 to it.
   Yesterday I placed a sum of money in the box and got a new amount.
   Today I put the new amount in the box and got $75 out.
   How much did I put in the box yesterday?

8. Play this game with a partner.
Each of you needs an octahedron and a cube like these:
The faces of a red octahedron are labelled from +1 to +8.
The faces of a white cube are labelled from +1 to +6.
Take turns to roll the two solids.
Subtract the red number from the white number.
The person with the lesser number scores a point.
The first person to reach 20 points is the winner.

9. On your first birthday, you have 1 candle on your cake.
On your second birthday, you have 2 candles on your cake, and so on, every year.
How many candles will be needed to celebrate your first 16 birthdays?

10. A radio station plays an average of 16 songs every hour.
One-half the songs are pop, one-quarter are jazz, one-eighth are country, and the rest are classical.
One show is 3 h long. The songs are played at random.
a) How many classical songs would be played?
b) What is the probability that the first song played is jazz?

11. An octahedron has eight faces labelled 1 to 8.
A cube has six faces labelled 1 to 6.
a) Both solids are rolled. What is the probability that the sum of the numbers is 8?
b) Both solids are rolled. What is the probability that the sum of the numbers is a prime number?

Empty the Rectangles

Game

YOU WILL NEED

2 number cubes labelled 1 to 6; 12 counters

NUMBER OF PLAYERS

2

GOAL OF THE GAME

To remove all counters from all rectangles

What strategies can you use to improve your chances of winning this game?

HOW TO PLAY THE GAME:

1. Each player draws 6 rectangles on a piece of paper.

Label each rectangle from 0 to 5.

2. Each player places her 6 counters in any or all of the rectangles.
You can place 1 counter in each rectangle, or 2 counters in each of 3 rectangles, or even 6 counters in 1 rectangle.

3. Take turns to roll the number cubes.
Find the difference of the numbers.
You remove counters from the rectangle labelled with that number.
For example, if you roll a 6 and a 4, then $6 - 4 = 2$; so, remove all counters from rectangle 2.

4. The winner is the first person to have no counters left in any rectangle.

Math Link

Sports

You know that a batting average of 0.300 means that a player has an average of 3 hits in 10 at bats. Research other examples of relative frequency in sport. Write what you find out.

Unit Review

Review any lesson with etext online tutorial

What Do I Need to Know?

- ☑ Relative frequency $= \dfrac{\text{Number of times an outcome occurs}}{\text{Number of times experiment is conducted}}$
- ☑ Theoretical probability $= \dfrac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$

What Should I Be Able to Do?

For extra practice, go to page 448.

LESSON

11.1

1. **a)** Use a tree diagram to show the possible combinations for a breakfast. You can choose:
- a banana, an orange, or an apple
- carrots, celery, or cucumber,

and
- yogurt or cheese

b) How many outcomes have a banana and cheese?

c) How many outcomes have an orange, celery, or a yogurt?

d) How many outcomes do not have an apple?

2. Four coins are tossed.

a) List all the possible outcomes.

b) How many outcomes have exactly 1 head?

c) How many outcomes have exactly 2 tails?

d) How many outcomes have at least 3 tails?

11.2

3. A biologist tested a new vaccine. She found that in 500 trials, the test was successful 450 times.

a) What is the relative frequency that the vaccine is successful?

b) Suppose the vaccine is used on 15 000 people. How many successes can be expected?

4. The owner of a shop recorded customer sales for one week.

Gender	Purchase	No Purchase	Total
Male	125	65	190
Female	154	46	200

Determine the relative frequency of each outcome.

a) A customer is male.

b) A customer is female.

c) A customer makes a purchase.

d) A male does not make a purchase.

e) A female makes a purchase.

LESSON

5. Is each statement true or false? Explain.

a) When a coin is tossed 100 times, it will never show tails 100 times.

b) When a coin is tossed 100 times, it is unlikely to show heads 100 times.

c) When a coin is tossed 100 times, it will show tails exactly 50 times.

d) The more often a coin is tossed, the more likely that $\frac{1}{2}$ the results will be tails.

11.3 **6.** In a game show, each contestant spins the wheel once to win the money shown.

a) Are the probabilities of winning the amounts equally likely? Explain.

b) What is the probability of winning $100?

c) What is the probability of winning less than $50?

d) What is the probability of winning from $30 to $70?

7. Twenty cards are numbered from 1 to 20. The cards are shuffled. A card is drawn. Find the probability that the card has:

a) an odd number

b) a multiple of 4

c) a number that is not a perfect square

d) a prime number

11.4 **8.** Each of the numbers 1 to 15 is written on a separate card. The cards are shuffled and placed in a pile face down. A card is picked from the pile. Its number is recorded. The card is returned to the pile. In 99 trials, about how many times would you expect each outcome?

a) a 6

b) a multiple of 3

c) a number less than 10

d) an even number

9. What is the chance of each outcome?

a) tossing 2 coins and getting:

i) 2 heads

ii) 1 tail and 2 heads

b) tossing 3 coins and getting:

i) 1 head and 2 tails

ii) at least 1 tail

10. An electronic game has three coloured sectors. A colour lights up at random, followed by a colour lighting up at random again. What is the chance the two consecutive colours are the same?

Practice Test

1. A theatre shows movies on Saturday and Sunday.
There are matinee and evening shows.
There are adult, child, and senior rates.
Draw a tree diagram to show the possible ticket types.

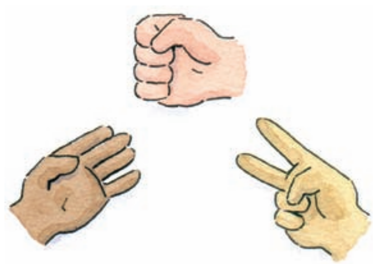
2. A number cube is labelled 1 to 6.
The cube is rolled 60 times.
Predict how many times each outcome will occur.
Explain each answer.

a) 1 is rolled.
b) An even number is rolled.
c) A number greater than 3 is rolled.
d) 9 is rolled.

3. **a)** In Sarah's first 30 times at bat, she had 9 hits.
What is Sarah's batting average?
b) In Sarah's next game, she had 3 hits in 4 times at bat.
What is Sarah's new batting average?
c) How many hits would you expect Sarah to have in 90 times at bat? Explain your reasoning.

4. A number cube is labelled 1 to 6.
Suppose you roll a number cube twice.
Is it more likely you will get a 3 then a 5, or a 3 then a 3?
Explain your reasoning.

5. In the game "rock, paper, scissors," 2 players make hand signs.
Players can make a hand sign for rock, paper, or scissors.
On the count of 3, players show their hand signs.
Suppose the players choose their signs at random.
In 75 games, how many times would you expect to see both players showing rock?



Unit Problem

Games of Chance

Part 1

Emma and Jonah created a spinner game called Match/No-Match.
Two people play the game.
A player spins the pointer twice.

If the pointer lands on the same colour (a match), the player scores.
If the pointer lands on different colours (a no-match), the opponent scores.

Jonah and Emma reasoned that, since there are two matching combinations (red/red and green/green), a player should score only 1 point for a match, and the opponent should score 2 points for a no-match.

Play the Match/No-Match game. Take at least 50 turns each.

Use the results to find the relative frequency of a match and of a no-match.

List the possible outcomes of a turn (two spins).
Find the theoretical probability of a match and a no-match.

Do you think the players have equal chances of winning? Explain.

Part 2

Design a game using spinners, number cubes, coins, or any other materials.
The game should use two different items.

Play the game.

Do the players have equal chances of winning? Explain.
Calculate some probabilities related to your game.
Show your work.

Check List

Your work should show:

- ✓ all calculations of frequency or probability, in detail
- ✓ diagrams, tables, or lists to show possible outcomes and results of each game
- ✓ an explanation of players' chances of winning each game
- ✓ correct use of the language of probability

Reflect on the Unit

Give at least two examples of how you use probability in everyday life.